CHAPTER 7

Decision-Making under Risk and Uncertainty

7.1 Introduction

The theory of expected utility combines the concept of utility from Chapter 2 with the concept of probability from Chapter 4 into an elegant and powerful theory of choice under risk. The resulting theory, which we explored in the previous chapter, is widely used. Yet there are situations in which people fail to conform to the predictions of the theory. In addition, there are situations in which it is seemingly rational to violate it. In this section we explore some such situations. We will also continue to explore what behavioral economists do in the face of systematic deviations from standard theory. To capture the manner in which people actually make decisions under risk, we will make more assumptions about the value function, which we first came across in Section 3.5, and introduce the probability-weighting function. Both these functions are essential parts of prospect theory, the most prominent behavioral theory of choice under risk.

7.2 Framing effects in decision-making under risk

For the next set of problems, suppose that you are a public health official.

Example 7.1 Asian disease problem 1 Imagine that the US is preparing for the outbreak of an unusual Asian disease, which is expected to kill 600 people. Two alternative programs to combat the disease have been proposed. Assume that the exact scientific estimate of the consequences of the programs are as follows: if Program A is adopted, 200 people will be saved; if Program B is adopted, there is 1/3 probability that 600 people will be saved, and a 2/3 probability that no one will be saved. Which of the two programs would you favor?

When this problem was first presented to participants, 72 percent chose A and 28 percent chose B.

Example 7.2 Asian disease problem 2 Imagine that the US is preparing for the outbreak of an unusual Asian disease, which is expected to kill 600 people. Two alternative programs to combat the disease have been proposed. Assume that the exact scientific estimate of the consequences of the programs are as follows: if Program C is adopted 400 people will die; if Program D is adopted there is 1/3 probability that nobody will die, and 2/3 probability that 600 people will die. Which of the two programs would you favor?

When this problem was first presented to participants, only 22 percent chose C and 78 percent chose D.

The observed response pattern is puzzling. Superficial differences aside, option A is the same as option C, and option B is the same as option D.

Before discussing what may be going on, let us briefly explore why this response pattern is hard to reconcile with expected-utility theory. As we learned in the previous chapter, expected-utility theory does not by itself say whether you should choose the safe or the risky option: the theory does not specify what your risk preference should be. But the theory does say that your choice should reflect your utility function. What matters is whether the point marked X in Figure 7.1 falls above or below the utility function itself. If the X falls below the curve, you will choose the safe option. This will occur if your utility function is concave, like the curve marked 1, and you are risk averse. If the X falls above the curve, you will choose the gamble. This will occur if your utility function is convex, like the curve marked 2, and you are risk prone. If the X falls on the curve, you are indifferent between the two options. This will occur if your utility function is a straight line, like the dashed line in the figure. The point is that as long as you act in accordance with expected-utility theory, you will prefer the safe option no matter how it is described, or you will prefer the risky option no matter how it is described, or you will be indifferent between the two. Your preference should definitely not depend on how the options are described.

So how do we account for the behavior exhibited in the study above? The key is to notice that the behavior can be interpreted in terms of framing. As you will recall from Section 3.5, framing effects occur when preferences and behavior are responsive to the manner in which the options are described, and in particular to whether the options are described in terms of gains or in terms of losses. Options A and B are both framed in a positive way, in terms of the lives that might be saved; that is, they are presented in a gain frame. Options C and D are both framed in a negative way, in terms of the lives that might be lost; that is, they are presented in a loss frame.

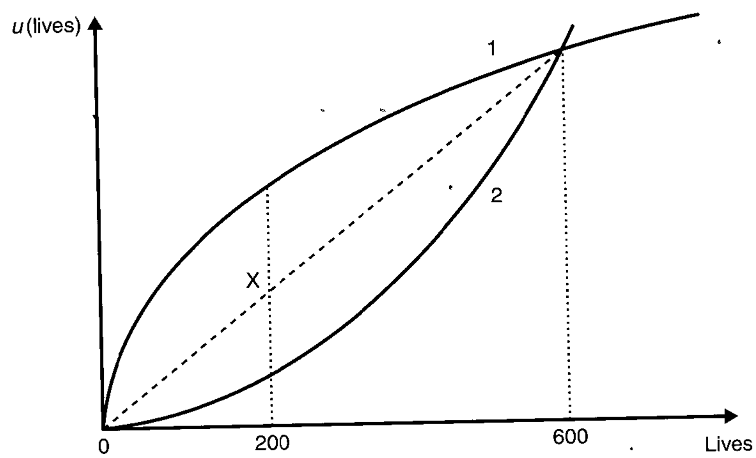

Figure 7.1 The utility of human lives

The talk about gains versus losses may remind you of our acquaintance the value function from prospect theory, which we used to model framing effects in Section 3.5. There, we learned that unlike the utility function, which ranges over *total* endowments, the value function ranges over *changes* in the endowment. We already know that many behavioral phenomena can be modeled by assuming a value function that is steeper for losses than for gains. Now we add the assumption that the value function has different curvatures for losses and for gains. In the realm of losses, we assume that the curve bends upwards (when moving from left to right), so that people are risk prone; in the realm of gains, we assume that the curve bends downwards, so that people are risk averse. In other words, the value function is convex in the realm of losses and concave in the realm of gains. This generates an S-shaped value function, as shown in Figure 7.2.

The curvature of the value function has interesting implications. For one thing, it entails that the absolute difference between $v(\pm 0)$ and $v(+10)$ is greater than than the absolute difference between $v(+1000)$ and $v(+1010)$, and that this is true for losses as well as for gains. We can establish the result algebraically, as the next example shows.

Example 7.3 Curvatures An S-shaped value function $v(\cdot)$ can be defined by an expression that has two components: one corresponding to the realm of gains and one corresponding to the realm of losses. For example:

$$v(x) = \begin{cases} \sqrt{x/2} & \text{for gains } (x \geq 0) \\ -2\sqrt{|x|} & \text{for losses } (x < 0) \end{cases}$$

Using this equation, the value of ± 0 is $v(\pm 0) = 0$ while the value of $+10$ is $v(+10) = \sqrt{10/2} = \sqrt{5} \approx 2.24$. The difference is $v(+10) - v(\pm 0) \approx 2.24 - 0 = 2.24$. Meanwhile, the value of $+1000$ is $v(+1000) = \sqrt{+1000/2} \approx 22.36$ while the value of $+1010$ is $v(+1010) = \sqrt{+1010/2} \approx 22.47$. The difference is only $v(+1010) - v(1000) \approx 22.47 - 22.36 = 0.11$. The difference between $v(\pm 0)$ and $v(+10)$ is much greater than than the difference between $v(+1000)$ and $v(+1010)$.

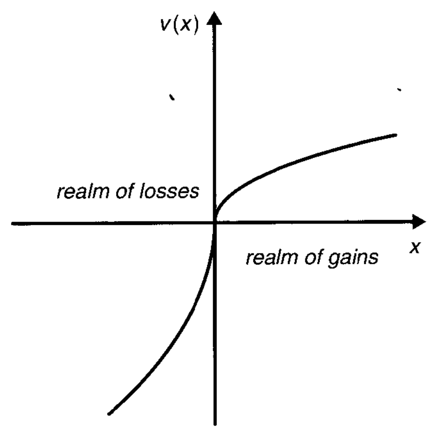

Figure 7.2 The value function

Because we are dealing with positive numbers throughout, we are in the realm of gains, which means that we are using the upper half of the equation. In the next exercise you will be using the lower half of the equation, since you will be dealing with negative numbers and the realm of losses. By the way, $|x|$ is the absolute value of x: that is, x with the minus sign removed (if there is one).

Exercise 7.4 Curvatures, cont. Given the same value function, which is greater: the absolute difference between $v(\pm 0)$ and $v(-10)$ or the absolute difference between $v(-1000)$ and $v(-1010)$?

Exercise 7.5 Jacket/calculator problem, again Consider again the classic jacket/calculator example from Section 3.2. Recall that many people were willing to make the drive when they could save \$5 on a \$15 calculator but not when they could save \$5 on a \$125 calculator. Using the same value function, show that the difference between $v(+10)$ and $v(+15)$ is much greater than than the difference between $v(+120)$ and $v(+125)$.

The exercise shows that an S-shaped value function can in fact account for the observed behavior in the jacket/calculator case.

We are now in a position to return to the Asian disease problem. The assumption that the value function is convex in the realm of losses and concave in the realm of gains helps account for the behavior of people facing this problem. The essential insight is that participants presented with the gain frame (as in Example 7.1) take their reference point to be the case in which no lives are saved (and 600 lost); participants presented with the loss frame (as in Example 7.2) take their reference point to be the case in which no lives are lost (and 600 saved). We can capture this in one graph by using two value functions to represent the fact that the two groups of participants use different outcomes as their reference point. In Figure 7.3, the concave value function on the top left belongs to people in the gain frame, while the convex value function on the bottom right belongs to people in the loss frame. As you can tell

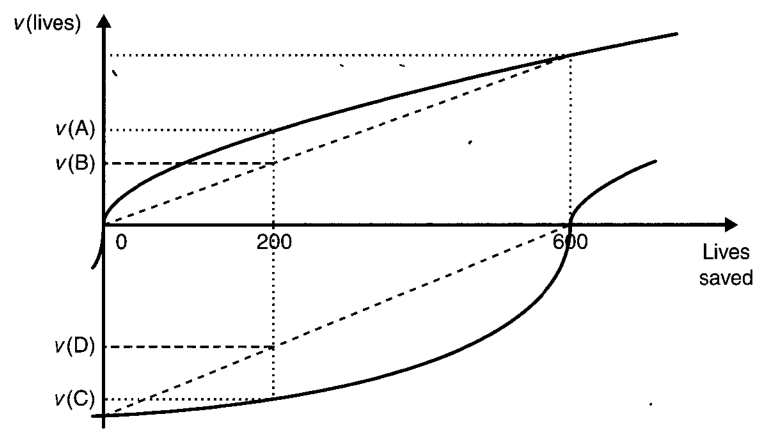

Figure 7.3 The value of human lives

from the figure, people in the gain frame will prefer A to B, but people in the loss frame will prefer D to C.

Exercise 7.6 The ostrich farm Jen and Joe have an ostrich farm. They have just learned that the farm has been struck by an unusual virus. According to their vet, if they do nothing only 200 of the 600 animals will live. However, the vet offers an experimental drug. If this drug is used, the vet says there is a 2/3 chance that all animals will die, but a 1/3 chance that all animals will live. Jen says: "The drug isn't worth it. It's better to save 200 animals for sure than risk saving none." Joe says: "I think we should use the drug. Even if it's risky, that's the only way we have a chance of losing no animals at all. Taking the risk is better than losing 400 animals for sure." Draw a graph explaining how the two can come to such different realizations even though they have value functions with the same shape.

The following example is another nice illustration of the phenomenon.

Example 7.7 Prospect evaluation Consider the following two problems:

(a) In addition to whatever you own, you have been given \$1000. You are now asked to choose between (A) a 50 percent chance of winning \$1000 and (B) winning \$500 for sure.

(b) In addition to whatever you own, you have been given \$2000. You are now asked to choose between (C) a 50 percent chance of losing \$1000 and (D) losing \$500 for sure.

In terms of final outcomes, (A) is obviously equivalent to (C) and (B) to (D). Yet 84 percent of participants chose B in the first problem, and 69 percent chose C in the second.

The difference between Example 7.7(a) and (b) is that in the former, outcomes are described in a gain frame, whereas in the latter, they are described in a loss frame. Consequently, in (a) the gamble represents an opportunity to win the big prize, whereas in (b) the gamble represents an opportunity to prevent a loss. To show how the observed response pattern might emerge we can analyze the problem algebraically.

Exercise 7.8 Prospect evaluation, cont. This exercise refers to Example 7.7 above. Suppose that your value function $v(\cdot)$ is defined by: $v(x) = \sqrt{x/2}$ for gains $(x \geq 0)$ and $v(x) = -2\sqrt{|x|}$ for losses $(x < 0)$

(a) Draw the curve for values between −4 and +4. Confirm that it is concave in the domain of gains and convex in the domain of losses.

(b) Assuming that you have integrated the \$1000 into your endowment, what is the value of (A)?

(c) Assuming that you have integrated the \$1000 into your endowment, what is the value of (B)?

(d) Assuming that you have integrated the \$2000 into your endowment, what is the value of (C)?

(e) Assuming that you have integrated the \$2000 into your endowment, what is the value of (D)?

Notice how (B) turns out to be better than (A), but (C) better than (D).

The idea that people are risk averse in the domain of gains but risk prone in the domain of losses helps explain a range of phenomena. It can explain why some people are unable to stop gambling: once they find themselves in the red – which they will, soon enough, when playing games like roulette – they enter the domain of losses, where they are even more risk prone than they used to be. There is evidence that people betting on horses, etc., are more willing to bet on long shots at the end of the betting day. This phenomenon is often accounted for by saying that people who have already suffered losses are more prone to risk-seeking behavior. Analogously, the idea can explain why politicians continue to pursue failed projects and generals continue to fight losing wars: as initial efforts fail, the responsible parties enter the domain of losses, in which they are willing to bet on increasingly long shots and therefore take increasingly desperate measures. Somewhat paradoxically, then, this analysis suggests that people, countries, and corporations can be expected to be most aggressive when they are weakest – not when they are strongest, as you might think.

Here are some more exercises.

Exercise 7.9 A person's value function is $v(x) = \sqrt{x/2}$ for gains and $v(x) = -2\sqrt{|x|}$ for losses. The person is facing the choice between a sure \$2 and a 50–50 gamble that pays \$4 if she wins and \$0 if she loses.

(a) Show algebraically that this person is loss averse, in the sense that she suffers more when she loses \$4 than she benefits when she receives \$4.
(b) If she takes the worst possible outcome (\$0) as her reference point, what is the value of the sure amount and the gamble? Which would she prefer?
(c) If she takes the best possible outcome (\$4) as her reference point, what is the value of the sure amount and the gamble? Which would she prefer?

Exercise 7.10 Another person with the same value function is facing the choice between a sure \$2 and a 50–50 gamble that pays \$5 if he wins and \$1 if he loses.

(a) If he takes the worst possible outcome as his reference point, what is the value of the sure amount and the gamble? Which would he prefer?
(b) If he takes the best possible outcome as his reference point, what is the value of the sure amount and the gamble? Which would he prefer?

Exercise 7.11 Relative income It is well known that poor people, who can least afford to play the lottery, are most likely to do so. In a 2008 study, researchers wanted to know whether manipulating people's perceptions of their income can affect their demand for lottery tickets. Half of the participants were made to feel rich by answering a question about their yearly income on a scale from "<\$10k," "\$10k–\$20k," and so on, to ">\$60k." The other half were made to feel poor by answering the same question on a scale from "<\$100k," "\$100k–\$200k," and so on, to ">\$1M." At the conclusion of the study, participants who were made to feel relatively poor were more likely to choose lottery tickets than cash as a reward for their participation. Are these findings consistent with the analysis in this section or not?

Framing effects should not be confused with wealth effects, which occur when people's risk aversion changes when they go from being poor to being rich (or the other way around), and which can be represented using a single utility function. It would be normal, for instance, if your curve got flatter and flatter as your wealth increased, thereby making you less risk averse. Not all the data can be easily accommodated in this framework, however; much of it is better explained by a value function that is convex in the realm of losses and concave in the realm of gains. In the following section, we discuss other applications of these ideas.

7.3 **Bundling and mental accounting**

The fact that the value function is concave in the domain of gains and convex in the domain of losses has other interesting implications, one being that it matters how outcomes are **bundled** (see Section 3.5). Suppose that you buy two lottery tickets at a charity event, and that you win \$25 on the first and \$50 on the second. There are different ways to think of what happened at the event (see Figure 7.4). You can **integrate** the outcomes, and tell yourself that you just won \$75, which in value terms would translate into $v(+75)$. Or you can **segregate** the outcomes and tell yourself that you first won \$25 and then won \$50, which in value terms would translate into $v(+25) + v(+50)$. Bundling can be seen as an instance of framing: at stake is whether you frame the change as one larger gain or as two smaller gains.

According to the standard view, bundling should not matter. The utility function ranges over total endowments, and no matter how you describe the various outcomes you end up with an additional \$75 dollars in your pocket.

Figure 7.4 Integration vs. segregation of gains. Illustration by Cody Taylor

In utility terms, then, if you start off at $u(w)$, you will end up at $u(w + 25 + 50)$ $= u(w + 75)$ either way.

According to prospect theory, however, bundling matters. Suppose that you start off with wealth w and that you take the status quo as your reference point. When the two gains are integrated, the value of winning $75 can be characterized as in Figure 7.5(a). However, when the two gains are segregated, the situation looks different. This is so because you have the time to adjust your reference point before assessing the value of the second gain. When the two gains are segregated, your picture will look more as in Figure 7.5(b), where the dashed line represents your value function relative to the new reference point. It should be clear just from looking at these two figures that the value of a $25 gain plus the value of a $50 gain is greater than the value of a $75 gain; that is $v(+25) + v(+50) > v(+75)$. This result follows from the value function being

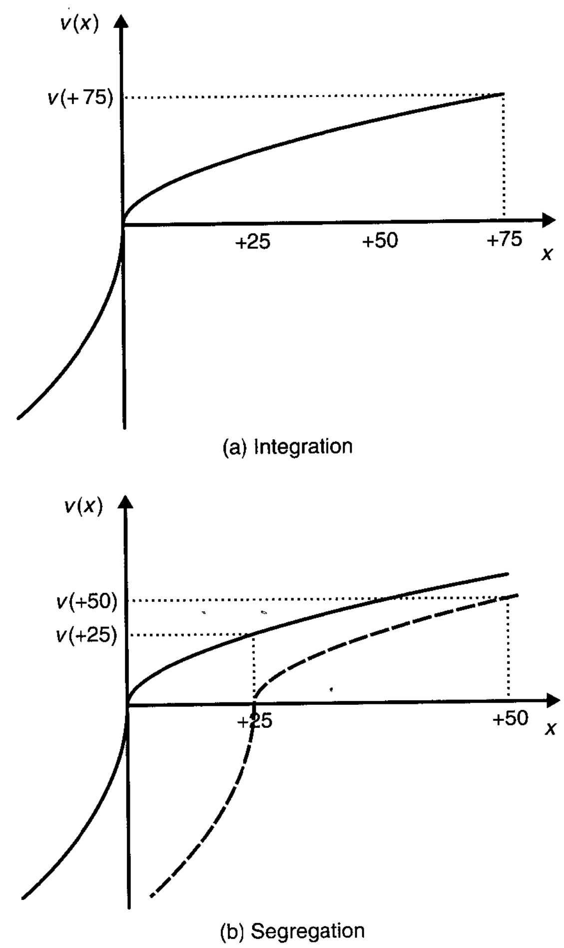

Figure 7.5 Evaluation of two gains

concave in the domain of gains. The upshot is that people value two gains more when they are segregated than when they are integrated.

An analogy might help. If you are in a totally dark room and you turn on a light bulb, there is a huge difference: you can see, which is wonderful if you would rather not be in the dark. If you add a second light bulb of the same wattage, you will experience a small change in brightness, but the difference is not going to be that large. It is certainly not going to be as large as going from zero light bulbs to one: going from one to two makes a much smaller difference than going from zero to one. Something similar occurs with money. Winning a small amount is good. Winning ten times that amount is much better, obviously, but it is not ten times as good. As a result, ten small gains are experienced as more impressive than the one gain that is ten times as large.

The fact that gains are valued more when segregated helps explain a variety of phenomena. For example, it explains why people do not put all their Christmas presents in one big box, even though that would save them time and money on wrapping: the practice of wrapping each present separately encourages the recipient to segregate the gains. The analysis also suggests that it is even better to give separate presents on separate nights, as on Hanukkah, rather than delivering all presents on Christmas, since this would do even more to encourage recipients to separate the gains. The analysis indicates that it is better to give people multiple small presents over the course of the year than to give them one big present once a year. While it is in good taste to give your spouse a present on your anniversary, you may wish to save some of the money and buy smaller presents during the rest of the year too. Similarly, segregation explains why fancy meals are served up dish-by-dish, rather than all at one time: chances are the eater will enjoy it more that way. In addition, the effect of segregating gains explains why workers receive end-of-year bonuses: receiving a \$50k salary plus a \$5k bonus encourages the segregation of gains in a manner that receiving a \$55k salary does not. Finally, the value of segregated gains explains why people on daytime television try to sell pots and pans by offering to throw in lids, knives, cutting boards, and so on, rather than simply offering a basket consisting of all these goods. Again, this practice encourages the segregation of the gains.

Exercise 7.12 Evaluation of gains Yesterday, you had a decent day: you first received a \$48 tax refund, and then an old friend repaid a \$27 loan you had forgotten about. Suppose that your value function $v(\cdot)$ is defined by: $v(x) = \sqrt{x/3}$ for gains $(x \geq 0)$ and $v(x) = -3\sqrt{|x|}$ for losses $(x < 0)$

(a) If you integrate the two gains, what is the total value?
(b) If you segregate the two gains, what is the total value?
(c) From the point of view of value, is it better to integrate or to segregate?

Meanwhile, people are less dissatisfied when multiple losses are integrated than when they are segregated. By constructing graphs like those in Figure 7.5, you can confirm that, from the point of view of value, a \$25 loss plus an additional \$50 loss is worse than a \$75 loss; that is, $v(-25) + v(-50) < v(-75)$.

This result follows from the fact that the value function is convex in the domain of losses. Notice that if you do purchase the pots and pans from daytime television, and get all the other stuff in the bargain, your credit card is likely to be charged only once, meaning that costs are integrated. This is no coincidence: by encouraging customers to integrate the losses while segregating the gains, marketers take maximum advantage of these effects.

Example 7.13 Stalin Soviet dictator Joseph Stalin is alleged to have said: "The death of one man is a tragedy; the death of millions is a statistic." This line captures an important insight about integration: a million deaths is nowhere near as bad, from our subjective point of view, as a million times one death.

The fact that people experience less dissatisfaction when losses are integrated helps explain a variety of phenomena. It explains why sellers of cars, homes, and other pricey goods often try to sell expensive add-ons. Although you may never pay $1000 for a car radio, when it is bundled with a $25,999 car it may not sound like very much: for reasons explored in the previous section, a loss of $26,999 might not seem that much worse than a loss of $25,999. By contrast, since they are entirely different quantities, you might find it easy to segregate the car from the radio, which would further encourage you to accept the offer. Similarly, the wedding industry makes good use of integration by adding options that individually do not seem very expensive relative to what the hosts are already spending, but which jointly can cause major financial distress. "If you're spending $3000 on a dress, does it matter if you spend an additional $10 on each invitation?" The effects of integrating losses can also explain why so many people prefer to use credit cards rather than paying cash. When you use a credit card, though the monthly bill might be alarming, it only arrives once a month, thereby encouraging you to integrate the losses. By contrast, the satisfaction of receiving the things you buy is distributed throughout the month, thereby encouraging the segregation of gains.

Exercise 7.14 The opposite arrangement Suppose that the opposite were true: whenever you purchase something, you have to pay cash on the spot, but your purchases are not delivered until the end of the month in a giant box containing everything you bought in the last four weeks.

(a) Would you make more or fewer purchases this way?

(b) Use the language of integration and segregation to explain why.

Exercise 7.15 Air fares If you are old enough, you may remember the good old days when all sorts of conveniences were included in the price of an airline ticket. Under pressure to reduce the sticker price of their tickets, airlines have started charging less for the tickets themselves, but made it a habit to recover the losses by charging fees for everything from checked luggage to food and drink and early boarding. Perhaps they sell more tickets this way, but the effort is likely to sharply reduce customer satisfaction. Why?

This analysis might also explain why people hold on to cars even though taking cabs may be less expensive in the long run. The actual cost of owning a car for a month (including maintenance, car payments, insurance payments, gasoline, car washes, etc.) is so high that for many people it would make financial sense to sell the car and just hail cabs. The reason why people hesitate to do this (outside the major cities) may be related to the fact that car payments are made monthly or weekly, whereas taxi payments are made at the conclusion of every ride. Consequently, taxi companies could encourage more business by allowing people to run a tab and to pay it off once a month. The analysis can also explain why people prefer to pay a flat monthly fee for cell phone services, internet connections, and gym memberships: doing so permits them to integrate what would otherwise be many separate losses. The chances are you would not dream of joining a gym that charged you by the mile you ran on the treadmill. In part, this is for incentive-compatibility reasons: you do not want to join a gym that gives you a disincentive to exercise. But, in part, this may be because you wish to integrate the losses associated with your gym membership.

Exercise 7.16 Evaluation of losses Yesterday, you had a terrible day: you got a $144 speeding ticket on your way to the opera, and then had to pay $25 for a ticket you thought would be free. Suppose your value function remains that of Exercise 7.12.

(a) If you integrate the two losses, what is the total value?
(b) If you segregate the two losses, what is the total value?
(c) From the point of view of value, is it better to integrate or to segregate?

Example 7.17 Online booksellers If you look this book up at a large online retailer, you may see a little advertisement saying something like: "This book is frequently bought with Kahneman's *Thinking, Fast and Slow*. Buy both for twice the price!" Presumably the message contains no new information: you already knew that you could get more books for more money. How is this message supposed to work?

Related phenomena occur when experiencing a large loss in combination with a small gain or a small gain in combination with a large loss. People gain more value when they integrate a small loss with a large gain. That way, the pain of the loss is not felt so intensely. This phenomenon is referred to as **cancellation**. Suppose that you win a million dollars, but have to pay a 10 percent tax on your winnings. The theory suggests that you will derive more value from telling yourself that you won $900,000 than by telling yourself that you won $1,000,000 and then lost $100,000.

Exercise 7.18 The pain of paying taxes The previous paragraph suggests that how you feel about paying your taxes will depend on whether you integrate that cost with the money you made or not.

(a) If you are a politician known for favoring high taxes, should you encourage voters to integrate or segregate? How should you shape your message?
(b) If you are a politician known for favoring low taxes, should you encourage voters to integrate or segregate? How should you shape your message?

Meanwhile, people gain more value when they segregate a large loss from a small gain. The small gain is often described as a **silver lining.** This analysis explains why some cars, apartments, and other big-ticket items sometimes come with cash-back offers. A customer may be more likely to buy a car with a \$27k price tag and a \$1k cash-back offer than to buy the very same car with a \$26k price tag; the \$1k gain, when segregated, helps offset the pain associated with the \$27k loss. The analysis also explains why credit-card companies frequently offer reward points for spending money on the card. Though the value of the reward points is small relative to monthly fees and charges, the company hopes that you will segregate and that the reward points therefore will serve to offset the larger loss. Finally, silver-lining phenomena may explain why rejection letters frequently include lines about how impressed the committee was with your submission or job application. The hope is that disingenuous flattery, when segregated, will to some extent offset the much greater perceived loss of the publication or job.

Exercise 7.19 Silver linings For this question, suppose your value function is $v(x) = \sqrt{x/2}$ for gains and $v(x) = -2\sqrt{|x|}$ for losses. Last night, you lost \$9 in a bet. There was a silver lining, though: on your way home, you found \$2 lying on the sidewalk.

(a) If you integrate the loss and the gain, what is the total value?

(b) If you segregate the loss and the gain, what is the total value?

(c) From the point of view of value, is it better to integrate or to segregate?

When do people integrate and when do they segregate? One possibility that might come to mind is that people bundle outcomes so as to maximize the amount of value that they experience. This is called the **hedonic-editing hypothesis**. According to this hypothesis, people will (1) segregate gains, (2) integrate losses, (3) cancel a small loss against a large gain, and (4) segregate a small gain from a large loss. Unfortunately, data suggest that the hypothesis is not in general true. In particular, it seems, people frequently fail to integrate subsequent losses. If you think about it, the failure of the hedonic-editing hypothesis is unsurprising in light of the fact that we need parents, therapists, boyfriends, and girlfriends to remind us of how to think about things in order not to be needlessly unhappy.

Bundling may be driven in part by **mental accounting**: people's tendency, in their minds, to divide money into separate categories. Mental accounting can be helpful in that it may stop you from overspending on any one category. But mental accounting can itself cause people to overconsume or underconsume particular kinds of goods: if the mental "entertainment account" is seen as having money left in it, but the mental "clothing account" is seen as overdrawn, people might spend more on entertainment even though they would maximize utility by buying clothes. This kind of behavior violates **fungibility**: the idea that money has no labels. Mental accounting might also affect the manner in which goods are bundled. For example, coding goods as belonging to the same category is likely to encourage integration, whereas coding goods as belonging to separate categories is likely to encourage segregation.

7.4 The Allais problem and the sure-thing principle

The following decision problem is called the **Allais problem**.

Example 7.20 Allais problem Suppose that you face the following options, and that you must choose first between (1a) and (1b), and second between (2a) and (2b). What would you choose?

(1a) \$1 million for sure
(1b) An 89% chance of \$1 million & a 10% chance of \$5 million
(2a) An 11% chance of \$1 million
(2b) A 10% chance of \$5 million

A common response pattern here is (1a) and (2b). For the first pair, people may reason as follows: "Sure, \$5 million is better than \$1 million, but if I chose (1b) there would be some chance of winning nothing, and if that happened to me I would definitely regret not choosing the million dollars. So I'll go with (1a)." That is, a choice of (1a) over (1b) might be driven by regret aversion (see Section 6.2). For the second pair, people may reason in this way: "Certainly, an 11 percent chance of winning is better than a 10 percent chance of winning, but that difference is fairly small; meanwhile, 5 million dollars is a lot better than 1 million dollars, so I'll go with (2b)." For the second pair, the potential for regret is much less salient.

Unfortunately, this response pattern is inconsistent with expected-utility theory. To see this, consider what it means to prefer (1a) to (1b). It means that the expected utility of the former must be greater than the expected utility of the latter. Thus:

$$u(1M) > .89 * u(1M) + .10 * u(5M) \quad (7.1)$$

Preferring (2b) to (2a) means that the expected utility of the former must exceed the expected utility of the latter. Hence:

$$.10 * u(5M) > .11 * u(1M) \quad (7.2)$$

But because $.11 * u(1M) = (1 - .89) * u(1M) = u(1M) - .89 * u(1M)$, (7.2) is equivalent to

$$.10 * u(5M) > u(1M) - .89 * u(1M) \quad (7.3)$$

Rearranging the terms in (7.3), we get:

$$.89 * u(1M) + .10 * u(5M) > u(1M) \quad (7.4)$$

But (7.4) contradicts (7.1). So the choice pattern that we are analyzing is in fact inconsistent with expected-utility theory.

There is another way of seeing why the choice pattern is inconsistent with expected-utility theory. Suppose that you spin a roulette wheel with 100 slots: 89 black, 10 red, and 1 white. This permits us to represent the four options in table form, as in Table 7.1.

Table 7.1 The Allais problem

	Black (89%)	Red (10%)	White (1%)
(1a)	\$1M	\$1M	\$1M
(1b)	\$1M	\$5M	\$0
(2a)	\$0	\$1M	\$1M
(2b)	\$0	\$5M	\$0

Let us begin by considering the first decision problem: that between (1a) and (1b). The table reveals that when black occurs, it does not matter what you choose; you will get a million dollars either way. In this sense, the million dollars if black occurs is a **sure thing**. The expression $.89 * u(1M)$ appears in the calculation of the expected utility of (1a), of course, but because it also appears in the calculation of the expected utility of (1b), it should not affect the decision. Let us now consider the second decision problem. Again, the table reveals that when black occurs, you receive nothing no matter what you choose. So again, the \$0 is a sure thing and should not affect the relative desirability of (2a) and (2b). Thus, what happens in the column marked "Black" should not affect your choices at all. Instead, your choices will be determined by what happens in the other two columns. But once you ignore the column marked "Black," (1a) is identical to (2a) and (1b) is identical to (2b): just compare the two shaded areas in Table 7.1. So if you strictly prefer (1a), you are rationally compelled to choose (2a); if you strictly prefer (1b), you are rationally compelled to choose (2b).

The **sure-thing principle** says that your decisions should not be influenced by sure things. As this discussion indicates, it is implicit in expected-utility theory. The next exercise may help make the principle clearer.

Exercise 7.21 Sure-thing principle

(a) Suppose that you face the options in Table 7.2(a). Which state of the world does the sure-thing principle tell you to ignore?

(b) Suppose that you face the options in Table 7.2(b). What does the sure-thing principle tell you about this decision problem?

Exercise 7.22 Sure-thing principle, cont. Suppose that you face the options in Table 7.2(c) and that you must choose first between (1a) and (1b), and second between (2a) and (2b). What choice pattern is ruled out by the sure-thing principle?

As a normative principle, the sure-thing principle has its appeal, but it is not uncontroversial. Some people argue that violations of the sure-thing principle can be perfectly rational, and that there consequently is something wrong with expected-utility theory as a normative standard. Others insist that the sure-thing principle is a normatively correct principle. What is fairly clear, though, is that it is false as a description of actual behavior; people seem to violate it regularly and predictably. (We will return to this topic in the next section.)

One way to describe the Allais paradox is to say that people overweight outcomes that are certain, in the sense that they occur with a 100 percent probability. This tendency has been called the **certainty effect.** As suggested above, the certainty effect might result from regret aversion: whenever you forego a certain option for a risky one, there is some chance that you will experience regret. Thus, a desire to minimize anticipated regret would lead to the rejection of the option that is not certain.

Table 7.2 Sure-thing principle

	A	B	C
(1)	2	1	4
(2)	3	1	3

(a)

	P	Q	R	S
(1)	3	2	4	1
(2)	3	1	4	2

(b)

	X	Y	Z
(1a)	80	100	40
(1b)	40	100	80
(2a)	40	0	80
(2b)	80	0	40

(c)

The certainty effect is apparent in slightly different kinds of context as well, as the following example shows.

Example 7.23 Certainty effect Which of the following options do you prefer: (A) a sure win of \$30; (B) an 80 percent chance to win \$45? Which of the following options do you prefer: (C) a 25 percent chance to win \$30; (D) a 20 percent chance to win \$45?

In this study, 78 percent of respondents favored A over B, yet 58 percent favored D over C.

The observed behavior pattern is an instance of the certainty effect, since a reduction from 100 percent to 25 percent makes a bigger difference to people than a reduction from 80 percent to 20 percent.

Exercise 7.24 Certainty effect, cont. Show that it is a violation of expected-utility theory to choose (A) over (B) and (D) over (C) in Example 7.23. Notice that (C) and (D) can be obtained from (A) and (B) by dividing the probabilities by four.

Does the certainty effect appear in the real world? It might. In a study of 72 physicians attending a meeting of the California Medical Association, physicians were asked which treatment they would favor for a patient with a tumor, given the choice between a radical treatment such as extensive surgery (options A and C), which involves a greater chance of imminent death, and a moderate treatment such as radiation (options B and D). They were presented with the following options:

A 80 percent chance of long life
(20 percent chance of imminent death)
B 100 percent probability of short life
(0 percent chance of imminent death)
C 20 percent chance of long life
(80 percent chance of imminent death)
D 25 percent chance of short life
(75 percent chance of imminent death).

The certainty effect was plainly visible: in violation of expected-utility theory, 65 percent favored B over A, yet 68 percent favored C over D. The fact that medical doctors exhibit the same behavior patterns as other people should not surprise us. It might be helpful to know this, whether or not you are a medical doctor.

7.5 The Ellsberg problem and ambiguity aversion

The following decision problem is referred to as the **Ellsberg problem**. The problem is due to Daniel Ellsberg, a US military analyst otherwise famous for releasing the so-called Pentagon Papers. Ellsberg was the subject of the 2009 documentary *The Most Dangerous Man in America*.

Example 7.25 Ellsberg problem Suppose that Dan shows you an urn with a total of 90 balls in it. There are three kinds of ball: red, black, and yellow. You know (from a trustworthy authority) that 30 are red, but you do not know how many of the remaining 60 are black and how many are yellow: there could be anywhere from 0 black and 60 yellow to 60 black and 0 yellow. The composition of the urn is illustrated by Table 7.3.

Table 7.3 Dan's urn

	Red	Black	Yellow
Number of balls in urn	30	60	

Dan invites you to randomly draw a ball from the urn. He gives you the choic between two different gambles: (I) $100 if the ball is red, and (II) $100 dollars i the ball is black. Which one would you choose? Next, Dan gives you a choic between the following two gambles: (III) $100 if the ball is red or yellow, an (IV) $100 if the ball is black or yellow. Which one would you choose?

When faced with the Ellsberg problem, many people will choose (I) rath than (II), apparently because they know that the chances of winning are 1/3; they choose the other option the chances of winning could be anywhere fro 0 to 2/3. Meanwhile, many people will choose (IV) rather than (III), appa ently because they know the chances of winning are 2/3; if they choose th other option the chances of winning could be anywhere from 1/3 to 1.

However, and perhaps unfortunately, the choice of (I) from the first pair of options and (IV) from the second pair violates the sure-thing principle introduced in the previous section. The violation may be clearer if we represent the problem as in Table 7.4, which shows the payoffs for all four gambles and the three different outcomes.

As the table shows, what happens when a yellow ball is drawn does not depend on your choices. Whether you choose (I) or (II) from the first pair, when a yellow ball is drawn you will get nothing either way. The \$0 when a yellow ball is drawn is a sure thing. Whether you choose (III) or (IV) from the second pair, when a yellow ball is drawn you will get \$100 either way. Again, the \$100 is a sure thing. Thus, the sure-thing principle says that your choices should not depend on what happens when you draw a yellow ball. That is, your choice should not depend on what is going on in the last column of Table 7.4. Your choice must reflect your evaluation of what is going on in the two columns to the left only. Ignoring the column marked "Yellow," however, you will see that (I) and (III) are identical, as are (II) and (IV): just compare the two shaded areas in the table. Hence, unless you are indifferent, you must either choose (I) and (III) or (II) and (IV).

Table 7.4 The Ellsberg problem

	Red (R)	Black (B)	Yellow (Y)
I	100	0	0
II	0	100	0
III	100	0	100
IV	0	100	100

There is another way of showing how the choice pattern (I) and (IV) is inconsistent with expected-utility theory. A strict preference for (I) over (II) entails that $EU(\text{I}) > EU(\text{II})$, which means that:

$$\Pr(R) * u(100) + \Pr(B) * u(0) + \Pr(Y) * u(0) > \Pr(R) * u(0) + \Pr(B) * u(100) + \Pr(Y) * u(0)$$

Meanwhile, a strict preference for (IV) over (III) entails that $EU(\text{IV}) > EU(\text{III})$, which means that:

$$\Pr(R) * u(0) + \Pr(B) * u(100) + \Pr(Y) * u(100) > \Pr(R) * u(100) + \Pr(B) * u(0) + \Pr(Y) * u(100)$$

Let us assume that $u(0) = 0$ and that $u(100) = 1$, which is only to say that you prefer \$100 over nothing. If so, these two expressions imply that the following two conditions must simultaneously be satisfied:

$$\Pr(R) > \Pr(B)$$
$$\Pr(B) > \Pr(R)$$

But that is obviously impossible. So again, the choice pattern we have been talking about is inconsistent with expected-utility theory.

How do we explain the fact that people exhibit this inconsistency? The two rejected options – (II) and (III) – have something in common, namely, that the exact probability of winning is unclear. We say that these probabilities are **ambiguous**. By contrast, the favored options – (I) and (IV) – are not associated with ambiguous probabilities. The observed choices seem to reflect an unwillingness to take on gambles with ambiguous probabilities. We refer to this phenomenon as **ambiguity aversion**. Some people have a greater tolerance for ambiguity than others, but any aversion to ambiguity is a violation of expected-utility theory. Insofar as people are in fact ambiguity averse, which they seem to be, expected-utility theory fails to capture their behavior. And insofar as it is rationally permissible to take ambiguity into account when making decisions, expected-utility theory does not capture the manner in which people should make decisions.

Exercise 7.26 The coins Suppose that you have the opportunity to bet on the outcome of a coin toss. If the coin comes up heads, you win; if it comes up tails, you lose. Suppose also that you are ambiguity averse. Would you rather bet on a fair coin (with equal probabilities of coming up heads and tails) or on a loaded coin with unknown, unequal probabilities of coming up heads and tails?

Exercise 7.27 Tennis You have been invited to bet on one of three tennis games. In game 1, two extraordinarily good tennis players are up against each other. In game 2, two extraordinarily poor tennis players are up against each other. In game 3, one very good and one very bad player are up against each other, but you do not know which is good and which is bad. As a result, as far as you are concerned, the probability that any given player will win is 50 percent. Suppose that you are ambiguity averse. Which of the three games would you be *least* likely to bet on? Why?

There is no principled reason why people cannot be **ambiguity prone** rather than ambiguity averse. In fact, evidence suggests that people's behavior in the face of ambiguous probabilities depends on the context. According to the **competence hypothesis**, for example, people are less averse to ambiguity in contexts where they consider themselves particularly knowledgeable. Thus, a football fan may be ambiguity averse in the Ellsberg case (where outcomes are completely random) but ambiguity prone when predicting the outcomes of football games (where he or she feels like an expert).

Exercise 7.28 Nevada's boom and bust Las Vegas entrepreneur Andrew Donner does not gamble at the casinos. Instead, he invests in real estate in the city's downtown. Interviewed on *Marketplace*, Donner said: "Well, you know casinos, you somewhat know the odds, and I think there's something beautiful about being somewhat ignorant of your odds out in the business marketplace. You keep working and hopefully you win more than you lose."

(a) Is Donner ambiguity averse or ambiguity prone?
(b) Are his attitudes consistent with the competence hypothesis or not?

Even so, the Ellsberg paradox and ambiguity aversion have potentially vast implications. What they suggest is that people do not in general assign probabilities satisfying the axioms of the probability calculus to events with ambiguous probabilities. And in the real world, ambiguous probabilities are common. The probability of bankruptcies, oil spills, and nuclear meltdowns can be estimated, but, outside games of chance, some ambiguity almost always remains. Thus, it is highly likely that people's choices *do* reflect the fact that people are ambiguity averse – or prone, as the case may be. And perhaps choices *should* reflect the ambiguity of the probabilities too.

Example 7.29 Known knowns In a 2002 press briefing, American Defense Secretary Donald H. Rumsfeld said: "[As] we know, there are known knowns; there are things we know we know. We also know there are known unknowns; that is to say we know there are some things we do not know. But there are also unknown unknowns – the ones we don't know we don't know." Rumsfeld was roundly satirized following the briefing, but the distinctions he was trying to draw may be very important.

7.6 Probability weighting

The idea that the value function is concave in the domain of gains and convex in the domain of losses helps us to analyze a wide range of behaviors, as we saw in Sections 7.2 and 7.3. Yet there are widely observed behavior patterns that cannot be accommodated within this framework. Consider the fact that some people simultaneously gamble and purchase insurance. This is paradoxical from the point of view of expected-utility theory. If people are risk averse, they should buy insurance but not gamble; if they are risk prone, they should gamble but not buy insurance; and if they are risk neutral, they should do neither. It is theoretically possible that people have inverted-S-shaped utility functions, like the dashed line in Figure 6.9 on page 146, and that the inflection point (marked $x*$ in the figure) just happens to correspond to their present endowment. Yet it seems too much like a coincidence that this should be true for so many people.

Simultaneous gambling and insurance shopping are equally paradoxical from the point of view of the theory we have studied in this chapter so far. The fact that people are willing to accept a gamble in which they may win a large sum of money suggests that they are risk prone in the domain of gains, while the fact that they are willing to reject a gamble in which they may lose their house suggests that they are risk averse in the domain of losses. This would entail that their value function is convex in the domain of gains and concave in the domain of losses, which is the very opposite of what we have assumed to date. The only way to accommodate this behavior pattern within the framework above is to assume that people take the state when they win the grand prize as their reference point when gambling, and the state in which they lose things as their reference point when buying insurance. This seems artificial, however, in light of the other evidence that people otherwise frequently take their endowment as their reference point.

Another way to understand the observed behavior pattern is to think of those who gamble as well as those who buy insurance as prone to paying too much attention to unlikely events. The more weight you put on the probability of winning the lottery, the more likely you will be to gamble. And the more weight you put on the probability of losing your house, car, life, and limb, the more likely you will be to purchase insurance. This insight suggests a more systematic approach to explaining how people can simultaneously buy lottery tickets and insurance.

Prospect theory incorporates this kind of behavior by introducing the notion of **probability weighting**. We know from Definition 6.21 on page 142 that expected-utility theory says that the agent maximizes an expression of the following form:

$$EU(A_i) = \Pr(S_1) * u(C_{i1}) + \Pr(S_2) * u(C_{i2}) + \ldots + \Pr(S_n) * u(C_{in})$$

By contrast, prospect theory says that the agent maximizes an expression in which the value function $v(\cdot)$ is substituted for the utility function $u(\cdot)$, and in which probabilities are weighted by a **probability-weighting function** $\pi(\cdot)$.

Definition 7.30 Value *Given a decision problem as in Table 6.4 on page 136, the value (or weighted value) $V(A_i)$ of an act A_i is given by:*

$$\begin{aligned} V(A_i) &= \pi[\Pr(S_1)] * v(C_{i1}) + \pi[\Pr(S_2)] * v(C_{i2}) + \ldots + \pi[\Pr(S_n)] * v(C_{in}) \\ &= \sum_{j=1}^{n} \pi[\Pr(S_j)] v(C_{ij}). \end{aligned}$$

The probability-weighting function $\pi(\cdot)$ assigns weights, from zero to one inclusive, to probabilities. It is assumed that $\pi(0) = 0$ and that $\pi(1) = 1$. But, as shown in Figure 7.6, for values strictly between zero and one, the curve does not coincide with the 45-degree line. For low probabilities, it is assumed that $\pi(x) > x$, and for moderate and high probabilities, that $\pi(x) < x$.

The probability-weighting function can help resolve the paradox that some people simultaneously buy lottery tickets and insurance policies. A well-informed expected-utility maximizer weights the utility of winning the grand prize by the probability of winning it, which as we know from Section 4.4 is low indeed. Thus, expected-utility theory says that such outcomes should not loom very large in our decision-making. Prospect theory makes a different prediction. Winning the lottery and losing house, car, life, and limb are positive- but low-probability events, so the probability-weighting function implies that they would loom relatively large. And when such events loom large, people are willing to purchase lottery tickets and insurance policies. This helps explain why people fear airplane crashes, terrorist attacks, and many other unlikely things so much. Probability-weighting also explains why people purchase extended warranties on equipment such as computers, in spite of the fact that simple expected-value calculations suggest that for most people extended warranties are not a very good deal (see Exercise 6.19 on page 139).

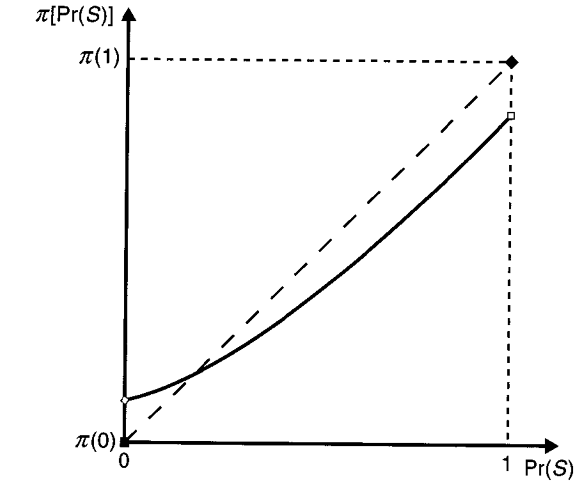

Figure 7.6 The probability-weighting function $\pi(\cdot)$

Table 7.5 Risk attitudes according to prospect theory

		Domain	
		Losses	**Gains**
Probability	Low	risk averse	risk prone
	Moderate	risk prone	risk averse
	High	risk prone	risk averse

Example 7.31 Freakonomics The book *Freakonomics* discusses the economics of crack-cocaine. Contrary to what many people think, the vast majority of crack dealers make little money – frequently less than the federally mandated minimum wage. They stay in the job, according to the *Freakonomics* authors, because of a small chance of joining the upper stratum of the organization, in which an exclusive "board of directors" makes decent money.

Even so, this does only seem to be part of the explanation. The directors do not make astronomical amounts of money, the probability of joining their group is small, and the probability of getting shot or landing in jail is high. We can augment the explanation by adding that aspiring directors might overweight the low probability of rising through the ranks and joining the board of directors. This is what prospect theory would predict.

As the authors suggest, the same analysis might apply to aspiring models, actors, concert pianists, and CEOs. The probability of succeeding in any one of these endeavors is low, yet people continue to bet that they will be the one who does. Their aspirations may, in part, be driven by the fact that they overweight the probability of success.

The probability-weighting function can also account for the certainty effect: the tendency to overweight outcomes that are certain (see Section 7.4). As Figure 7.6

Prospect theory

Prospect theory describes the process of making decisions as having two phases. During the **editing phase** the decision maker edits the options to facilitate assessment. Editing may involve a number of different operations, including:

- **Coding:** Describing outcomes as gains or losses as compared to some reference point, which may be the current endowment, somebody else's current endowment, or an expectation of a future endowment.
- **Combination:** Combining identical outcomes into a simpler one, so that a 25 percent chance of winning \$1 and another 25 percent chance of winning \$1 get represented as a 50 percent chance of winning \$1.
- **Simplification:** Simplifying the options, e.g., by rounding probabilities and outcomes. In particular, extremely small probabilities may be rounded down to zero and eliminated from consideration.

During the subsequent **evaluation phase** the decision maker assesses the edited options. The evaluation is based on two elements: the value function from Sections 3.5 and 7.2 and the probability weighting function from Section 7.6. The value function $v(\cdot)$ is S-shaped, meaning convex in the realm of losses and concave in the realm of gains (see Figure 7.2 on page 155). The probability-weighting function $\pi(\cdot)$ normally satisfies the following conditions: $\pi(0) = 0$ and $\pi(1) = 1$; otherwise for low probabilities $\pi(x) > x$ and for moderate and high probabilities $\pi(x) < x$ (see Figure 7.6 on page 173). The value (or weighted value) $V(A_i)$ of an act A_i is evaluated in accordance with the formula:

$$V(A_i) = \pi[\Pr(S_1)] * v(C_{i1}) + \pi[\Pr(S_2)] * v(C_{i2}) + \ldots + \pi[\Pr(S_n)] * v(C_{in})$$
$$= \sum_{j=1}^{n} \pi[\Pr(S_j)]v(C_{ij}).$$

In the special case when $\pi(x) = x$ and $v(x) = u(x)$, the value of an option equals its expected utility (see Definition 6.21 on page 142).

shows, there is a discontinuity at probabilities approaching one, so that as $x \rightarrow 1$, $\lim \pi(x) < \pi(1)$. Thus, events that are not certain (even when their probability is very high) will be underweighted relative to events that are certain.

The upshot is that people's behavior in the face of risk depends not just on whether outcomes are construed as gains or as losses relative to some reference point, but on whether or not the relevant probabilities are low. In the domain of losses, people tend to be risk prone, except for gambles involving a low-probability event of significant (negative) value, in which case they may be risk averse. In the domain of gains, people tend to be risk averse, except for gambles involving a low-probability event of significant (positive) value, in which case they may be risk prone. See Table 7.5 for a summary of these implications.

Example 7.32 Russian roulette Suppose that you are forced to play Russian roulette, but that you have the option to pay to remove one bullet from the loaded gun before pulling the trigger. Would you pay more to reduce the number of bullets in the cylinder from four to three or from one to zero? According to Kahneman and Tversky, if you are like most people, you would pay more to reduce the number from one to zero than from four to three. Why?

Reducing the number of bullets from four to three would reduce the probability of dying from 4/6 to 3/6. In this range, the probability-weighting function is relatively flat, meaning fairly unresponsive to changes in the underlying probability. Reducing the number of bullets from one to zero would reduce the probability of dying from 1/6 to 0. Here, there is a jump from $\pi(1/6) > 1/6$ to $\pi(0) = 0$. Thus, the value to you of reducing the number of bullets from one to zero exceeds that of reducing it from four to three.

Exercise 7.33 Lotteries as rewards Behavioral economists have found that using lotteries is an effective way to incentivize behavioral change. Thus, a person may be more likely to fill in a survey or take a pill if offered a lottery ticket with a 1/1000 probability of winning $1000 than if offered a cash payment of $1. This might seem counterintuitive, given that people are often risk averse. Use the probability-weighting function to explain why lottery tickets can be so appealing.

7.7 Discussion

In this chapter we have discussed situations in which people appear to violate the standard implicit in the theory of expected utility outlined in Chapter 6. The problem is not that people fail to maximize some mathematical utility function in their heads. Rather, the problem is that people's observed choices diverge from the predictions of the theory. Though the divergences are not universal, they are substantial, systematic, and predictable, and they can have real, and sometimes adverse, effects on people's decision-making. Behavioral economists take the existence of divergences to undercut the descriptive adequacy of the theory of expected utility. Obviously, the chapter does not purport to offer a complete list of violations of expected-utility theory. We have also discussed situations where people's firmly held intuitions about the rational course of action differ from the recommendations of expected-utility theory, as in the presence of ambiguous probabilities. This raises deep issues about the nature of rationality.

In addition, we have explored more theoretical tools developed by behavioral economists to capture the manner in which people actually make decisions. Among other things, we studied other components of prospect theory, including the S-shaped value function and the probability-weighting function. This concludes our review of prospect theory (see text box on page 174). These tools can be used not just to explain and predict but also to influence other people's evaluations. It appears that, under certain conditions, a person's risk preferences can be reversed simply by changing the frame of the relevant options. This gives behavioral economists more levers to use, which is obviously important for therapists, marketers, public health officials, and others who hope to affect people's behavior. But again, knowledge of the tools also permits us to anticipate and prevent other people from employing them.

In Section 6.6, we discussed the distinction between the rational and the right, and the fact that you cannot judge the rationality of a decision by examining the outcome alone. It will come as no surprise to hear that people do, in fact, often judge the rationality of people's decisions by the outcome. This is called **outcome bias**, and the phenomenon appears to be pervasive. Sports commentary offers many examples: coaches' decisions tend to be praised just in case they have the intended result independently of whether their decisions were rational in the first place. (Of course, the fact that a person's decisions often turn out to be right is evidence in favor of their rationality.) One person who understood outcome bias was the early sixth-century Roman philosopher Boethius, who noted: "[The] world does not judge actions on their merit, but on their chance results, and they consider that only those things which are blessed with a happy outcome have been undertaken with sound advice." Boethius knew about judgment. Once a powerful public official, he wrote these lines on death row having been convicted of sorcery and other charges. He was bludgeoned to death shortly thereafter, but the work, *The Consolations of Philosophy*, became one of the most widely read texts of the Middle Ages.

In Part 4 we will add another layer of complexity to the analysis, by introducing the topic of time.

ADDITIONAL EXERCISES

Exercise 7.34 Savings decisions You are lucky enough to have a million dollars in the bank. You have decided that there are only three serious investment options: putting it in your mattress, investing in stocks, and investing in bonds. Your utility function over total wealth is $u(x) = \sqrt{x}$. There is no inflation.

(a) If you stick your money in the mattress (where we can assume that it will be perfectly safe), how much utility will you have at the end of the year?

(b) Bonds are in general very dependable, but markets have been jittery as of late. You estimate that there is a 90 percent chance that you will gain 4 percent, but there is a 10 percent chance that you will gain nothing (zero percent). What is the expected utility of investing the \$1,000,000 in bonds?

ADDITIONAL EXERCISES cont.

(c) Stocks have recently been extremely volatile. If you invest in stocks, there is a 40 percent chance that you will gain 21 percent, a 40 percent chance that you will gain nothing (zero percent), and a 20 percent chance that you will lose ten percent. What is the expected utility of investing the $1,000,000 in stocks?

(d) Given that you are an expected-utility maximizer, what is the sensible thing to do with your money?

(e) If, instead of maximizing expected utility, you were exceedingly loss averse, what would you do?

Exercise 7.35 Zero expected value Prospect theory is consistent with the result that people frequently reject gambles with an expected value of zero. Suppose you are facing a gamble G with a 1/2 probability of winning $10 and a 1/2 probability of losing $10. According to prospect theory, would you prefer the gamble or the status quo? Assume that your value function is $v(x) = \sqrt{x/2}$ for gains and $v(x) = -2\sqrt{|x|}$ for losses.

Exercise 7.36 Life coaching Life coaches are people whose job it is to help you deal with challenges in your personal and professional life.

(a) If you spend too much money, life coaches will sometimes suggest that you cut up your credit cards and pay cash for everything. Use the language of integration and segregation to explain how this is supposed to help you rein in your spending.

(b) In order to help you spend money more wisely, life coaches sometimes suggest you allocate your money between a number of envelopes marked "food," "drink," "transportation," etc. What is the technical term for this kind of behavior?

(c) Though using the envelope technique may have real benefits, it can also lead to problems. Identify some possible negative consequences of this technique.

Exercise 7.37 Match each of the vignettes below with one of the following phenomena: *ambiguity aversion, cancellation, certainty effect, competence hypothesis, silver lining*, and *mental accounting*. If in doubt, pick the best fit.

(a) Abraham is seriously depressed after his girlfriend of several years leaves him for his best friend. His therapist tells him that every time a door closes, another one opens. Abraham thinks about all the other potential girlfriends out there and finally feels a little better.

(b) Berit is trying to save money by limiting the amount of money she spends eating out. She tells herself she must never spend more than $100 each week in restaurants. It is Sunday night, and Berit realizes that she has spent no more than $60 eating out during the past week. She does not quite feel like going out, but tells herself that she must not miss the opportunity to have a $40 meal.

(c) Charles is not much of a gambler and rarely accepts bets. The exception is politics. He considers himself a true policy expert and

ADDITIONAL EXERCISES cont.

is happy to accept bets when it comes to the outcome of local and national elections. His friends note that he still does not win the bets more than about half of the time.

(d) According to a common saying: "Better the devil you know than the devil you don't."

(e) Elissa very much wants to go to medical school, but cannot stand the thought of not knowing whether she will pass the rigorous curriculum. Instead, she decides to sign up for a less demanding physical therapy curriculum that she is confident that she can pass.

Problem 7.38 *Drawing on your own experience, make up stories like those in Exercise 7.37 to illustrate the various ideas that you have read about in this chapter.*

FURTHER READING

Framing effects and probability weighting, which are part of prospect theory, are discussed in Kahneman and Tversky (1979); see also Tversky and Kahneman (1981), the source of the Asian disease problem (p. 453). The example from the original prospect-theory paper appears in Kahneman and Tversky (1979, p. 273), and the two examples of the certainty effect in Tversky and Kahneman (1986, pp. S266–69); the study that manipulated people's perceptions of their income is Haisley et al. (2008). Bundling and mental accounting are explored in Thaler (1980) and (1985) and hedonic editing in Thaler and Johnson (1990). The Allais problem is due to Allais (1953); the certainty effect is discussed in Tversky and Kahneman (1986). The Ellsberg problem is due to Ellsberg (1961); the competence hypothesis is due to Heath and Tversky (1991). Donner is quoted in Gardner (2012) and Rumsfeld in US Department of Defense (2002). Probability-weighting is described in Kahneman and Tversky (1979); the roulette example (on p. 283) is attributed to Zeckhauser. *Freakonomics* is Levitt and Dubner (2005). Outcome bias is the topic of Baron and Hershey (1988); the lines from *The Consolations of Philosophy* appear in Boethius (1999 [*c* 524], p. 14).