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Reflections of Philosophy

PART

4

Intertemporal Choice

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9 Intertemporal Choice

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The Discounted Utility Model



8

8.1 Introduction

So far, we have treated decision problems as though all possible consequences of actions were instantaneous. That is, we have assumed that all relevant consequences occur more or less immediately, or at least at one and the same time. There are cases when this is a perfectly reasonable assumption. If you are playing roulette once and have preferences over money, for example, whatever happens will happen more or less at the same time.

Very often, however, time is a factor. When you decide whether to purchase the one-year warranty for your new tablet computer (see Exercise 6.19 on page 139), you are not only choosing between a sure option (investing in the warranty) and a risky option (foregoing the warranty), but you are also choosing between a certain loss now (since the warranty has to be paid for now) and the possibility of suffering a loss later (the possibility of having to pay to replace a broken tablet some time down the road).

It may be that most decisions have consequences that occur at different points in time. Some decisions have immediate benefits and deferred costs: procrastination, for example, is a matter of favoring some immediate good (a dinner and movie with friends) over some later benefit (a clean house). Other decisions have immediate costs and deferred benefits: savings behavior, for example, is a matter of favoring some later benefit (a comfortable retirement) over some immediate good (a new car). In this chapter and the next, we will talk about how to model decisions when time is a factor.

8.2 Interest rates

Before we begin the theory, let us talk about **interest**. Much of this should be familiar, but knowing how to think about interest is so useful that it justifies a review. Evidence from studies like that in Exercise 1.3 on page 9 indicates that many people have a limited grasp of how interest works.

Example 8.1 Interest Suppose you borrow \$100 for a year at an annual interest rate of 9 percent. At the end of the process, how much interest will you owe the lender?

The answer is $\$100 * 0.09 = \9 .

Slightly more formally, let r be the interest rate, P the **principal** (that is, the amount you borrow), and I the interest. Then:

$$I = Pr \quad (8.1)$$

This formula can be used to evaluate credit-card offers. Table 8.1 was adapted from a website offering credit cards to people with bad credit. Given this information, we can compute what it would cost to borrow a given amount by charging it to the card.

Table 8.1 Credit-card offers for customers with bad credit

Credit-card offer	APR	Fee
Silver Axxess Visa Card	19.92%	\$48
Finance Gold MasterCard	13.75%	\$250
Continental Platinum MasterCard	19.92%	\$49
Gold Image Visa Card	17.75%	\$36
Archer Gold American Express	19.75%	\$99
Total Tribute American Express	18.25%	\$150
Splendid Credit Eurocard	22.25%	\$72

Example 8.2 Cost of credit Suppose you need to invest \$1000 in a new car for one year. If you charge it to the Silver Axxess Visa Card, what is the total cost of the credit, taking into account the fact that you would be charged both interest and an annual fee?

Given that the Annual Percentage Rate (APR) $r = 19.92\% = 0.1992$, you can compute $I = Pr = \$1000 * 0.1992 = \199.20 . The annual fee is \$48. The total cost would be the interest (I) plus the annual fee: $\$199.20 + \$48 = \$247.20$.

Expressed as a fraction of the principal, this is almost 25 percent. And that is not the worst offer, as the next exercise will make clear.

Exercise 8.3 Cost of credit, cont. What would it cost to borrow \$1000 for one year using one of the other credit cards in Table 8.1? What if you need \$100 or \$10,000?

Fees and APRs fluctuate; never make decisions about credit cards without looking up the latest figures.

At the end of the year, the lender will want the principal back. In Example 8.1 above, the lender will expect the \$100 principal as well as the \$9 interest, for a total of \$109. Let L be the **liability**, that is, the total amount you owe the lender at the end of the year. Then:

$$L = P + I \quad (8.2)$$

Substituting for I from (8.1) in (8.2), we get:

$$L = P + I = P + (Pr) = P(1 + r) \quad (8.3)$$

It is sometimes convenient to define R as one plus r :

$$R = 1 + r \quad (8.4)$$

(8.1)

Together, (8.3) and (8.4) imply that:

$$L = PR \tag{8.5}$$

Returning to Example 8.1, we can use this formula to compute the liability: $L = \$100 * (1 + 0.09) = \109 , as expected. These formulas can also be used to compute interest rates given the liability and the principal.

Example 8.4 Implicit interest Suppose that somebody offers to lend you \$105 on condition that you pay them back \$115 one year later. What is the interest rate (r) implicit in this offer?

We know that $P = \$105$ and $L = \$115$. (8.5) implies that $R = L/P = \$115/\$105 = 1.095$. By (8.4), $r = R - 1 = 1.095 - 1 = 0.095$. Thus, the potential lender is implicitly offering you a loan at an annual interest rate of 9.5 percent.

Exercise 8.5 Payday loans Payday loan establishments offer short-term loans to be repaid on the borrower's next payday. Fees fluctuate, but such an establishment may offer you \$400 on the 15th of the month, provided you repay \$480 two weeks later. Over the course of the two weeks, what is the interest rate (r) implicit in this offer?

In some US states, the number of payday loan establishments exceeds the number of Starbucks and McDonald's locations combined. The answer to Exercise 8.5 suggests why. (See also Exercise 8.9.)

We can extend the analysis over longer periods of time. Broadly speaking, there are two kinds of scenario that merit our attention. Here is the first:

Example 8.6 Simple interest Imagine that you use a credit card to borrow \$100, and that every month the credit-card company will charge you an interest rate of 18 percent of the principal. Every month, you pay only interest, but you pay it off in full. At the end of the year, you also repay the \$100 principal. What is the total interest over the course of a year, expressed both in dollars and as a fraction of the principal?

Every month, by (8.1), you have to pay the credit-card company $I = Pr = \$100 * 0.18 = \18 in interest. Because you have to do this 12 times, the total interest will amount to $\$18 * 12 = \216 . As a fraction of the \$100 principal, this equals $\$216/\$100 = 2.16 = 216$ percent.

This is a case of **simple interest**. You can get the same figure by multiplying 18 percent by 12. Here is the other kind of case:

Example 8.7 Compound interest Imagine, again, that you use a credit card to borrow \$100 and that the monthly interest rate is 18 percent. In contrast to the previous example, however, you do not make monthly interest payments. Instead, every month your interest is added to the principal. What is the total interest over the course of a year, expressed both in dollars and as a fraction of the principal?

At the conclusion of the first month, by (8.3), given that $r = 0.18$, you will owe $L = P(1 + r) = \$100 * 1.18 = \118 . At the conclusion of the second month,

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th bad credit. Given this
o borrow a given amount

th bad credit

	Fee
o	\$48
%	\$250
o	\$49
%	\$36
%	\$99
%	\$150
%	\$72

o invest \$1000 in a new car
Visa Card, what is the total
you would be charged both

= 19.92% = 0.1992, you can
annual fee is \$48. The total
\$199.20 + \$48 = \$247.20.

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(8.2)

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your liability will be $L = \$118 * 1.18 = \139.24 . Notice that you could have gotten the same answer by computing $L = \$100 * 1.18 * 1.18 = \$100 * 1.18^2$. At the conclusion of the third month, your liability will be $L = \$100 * 1.18 * 1.18 * 1.18 = \$100 * 1.18^3 \approx 164.30$. At the conclusion of the 12th month, your liability will be $L = \$100 * 1.18^{12} \approx \728.76 . The liability at the end of the year includes the \$100 principal, so your interest payments, by (8.2), only add up to $I = L - P \approx \$728.76 - \$100 = \$628.76$. As a fraction of the principal, this equals $\$628.76/\$100 = 6.2876 = 628.76$ percent.

The answer to Example 8.7 is so much higher than the answer to Example 8.6 because the former involves **compound interest**. Unlike the case of simple interest, here the interest accumulated during the first period is added to the principal, so that you will end up paying interest on the interest accumulated during previous periods. Notice that in cases of compound interest, you cannot simply multiply the interest accumulated during the first period by the number of periods, as we did in the case of simple interest. Instead, with compound interest your liability after t periods is computed as:

$$L = PR^t \quad (8.6)$$

This formula gives us the answer to part (c) of Exercise 1.3, by the way: $\$200 * (1 + 0.10)^2 = \242 .

Albert Einstein is sometimes quoted as having said that compound interest is one of the most powerful forces in the universe. This would have been a wonderful quotation, had he actually said it, but there is no evidence that he did. Even so, you can get the power of compounding to work in your favor by saving your money and allowing the interest to accumulate.

Exercise 8.8 Savings Suppose that you put \$100 into a savings account today and that your bank promises a 5 percent annual interest rate.

- (a) What will your bank's liability be after 1 year?
- (b) After 10 years?
- (c) After 50 years?

Finally, let us return to the payday loan establishments.

Exercise 8.9 Payday loans, cont. Imagine that you borrow \$61 from a payday loan establishment. After one week, it wants the principal plus 10 percent interest back. But you will not have that kind of money; so, instead, you go to another establishment and borrow the money you owe the first one. You do this for one year. Interest rates do not change from one establishment to another, or from one week to another.

- (a) How much money will you owe at the end of the year?
- (b) What is the total amount of interest that you will owe at the end of the year, in dollar terms?
- (c) What is the total amount of interest that you will owe at the end of the year, expressed as a fraction of the principal?
- (d) What does this tell you about the wisdom of taking out payday loans?

Payday loan establishments have generated controversy, with some state legislatures capping the interest rates they may charge. Such controversy is not new. For much of the Christian tradition, it was considered a crime against nature to charge interest on loans, that is, to engage in **usury**. According to Dante's *Divina Commedia*, usurers are condemned to eternal suffering in the seventh circle of hell, in the company of blasphemers and sodomites; you know where Dante would have expected to find payday loan officers. On the other hand, payday loan establishments provide a service that (rational and well-informed) people may have reason to demand. Parents may be willing to pay a premium to have money for Christmas presents at Christmas rather than in January, for example.

Either way, as these exercises show, knowing the basics about interest rates can be enormously useful. Now, let us return the theory of decision.

8.3 Exponential discounting

Which is better, \$100 today or \$100 tomorrow? \$1000 today or \$1000 next year? The chances are you prefer your money today. There are exceptions – and we will discuss some of these in the next chapter – but typically people prefer money sooner rather than later. This is not to say that tomorrow you will enjoy a dollar any less than you would today. But it is to say that, *from the point of view of today*, the utility of a dollar today is greater than the utility of a dollar tomorrow.

There are many reasons why you might feel this way. The earlier you get your money, the more options will be available to you: some options may only be available for a limited period, and you can always save your money and go for a later option. In addition, the earlier you get your money, the longer you can save it, and the more interest you can earn. Whatever the reason, when things that happen in the future do not give you as much utility, from the point of view of today, as things that happen today, we say that you **discount the future**. The general term is **time discounting**. The extent to which you discount the future will be treated as a matter of personal preference, specifically, what we call **time preference**.

There is a neat model that captures the basic idea that people prefer their money sooner rather than later: the model of **exponential discounting**. Suppose that $u > 0$ is the utility you derive from receiving a dollar today. From your current point of view, as we established, the utility of receiving a dollar tomorrow is less than u . We capture this by multiplying the utility of receiving a dollar now by some fraction. We will use the Greek letter delta (δ) to denote this fraction, which we call the **discount factor**. Thus, from your current point of view, a dollar tomorrow is worth $\delta * u = \delta u$. As long as $0 < \delta < 1$, as we generally assume, this means that $\delta u < u$. Hence, today you will prefer a dollar today to a dollar tomorrow, as expected. From the point of view of today, a dollar the day after tomorrow will be worth $\delta * \delta * u = \delta^2 u$. Because $\delta^2 u < \delta u$, today you prefer a dollar tomorrow to a dollar the day after tomorrow, again as expected.

In general, we want to be able to evaluate a whole sequence of utilities, that is, a **utility stream**. Letting t represent time, we will use $t = 0$ to represent

today, $t = 1$ to represent tomorrow, and so on. Meanwhile, we let u_t denote the utility you receive at time t , so that u_0 represents the utility you receive today, u_1 represents the utility you receive tomorrow, and so on. We write $U^t(\mathbf{u})$ to denote the utility of stream \mathbf{u} from time t . The number we seek is the utility $U^0(\mathbf{u})$ of the entire utility stream $\mathbf{u} = \langle u_0, u_1, u_2, \dots \rangle$.

Definition 8.10 The delta function According to the delta function, the utility $U^0(\mathbf{u})$ of utility stream $\mathbf{u} = \langle u_0, u_1, u_2, \dots \rangle$ from the point of view of $t = 0$ is:

$$\begin{aligned} U^0(\mathbf{u}) &= u_0 + \delta u_1 + \delta^2 u_2 + \delta^3 u_3 + \dots \\ &= u_0 + \sum_{i=1}^{\infty} \delta^i u_i \end{aligned}$$

Thus, you evaluate different utility streams by adding the utility you would receive now, δ times the utility you would receive the next round, δ^2 times the utility you would receive in the round after that, and so on. The resulting model is called the **delta model**.

Utility streams can be represented in table form, as in Table 8.2. An empty cell means that the utility received at that time is zero. In order to compute the utility from the point of view of time zero, or any other point in time, you just need to know the discount factor δ . As soon as we are given the discount factor, we can use Definition 8.10 to determine which option you should choose.

Table 8.2 Simple time-discounting problem

	$t = 0$	$t = 1$	$t = 2$
a	1		
b		3	
c			4
d	1	3	4

Example 8.11 Exponential discounting Suppose that $\delta = 0.9$, and that each utility stream is evaluated from $t = 0$. If so, $U^0(\mathbf{a}) = u_0 = 1$, $U^0(\mathbf{b}) = \delta u_1 = 0.9 * 3 = 2.7$, $U^0(\mathbf{c}) = \delta^2 u_2 = 0.9^2 * 4 = 3.24$, and $U^0(\mathbf{d}) = u_0 + \delta u_1 + \delta^2 u_2 = 1 + 2.7 + 3.24 = 6.94$. Hence, if given the choice between all four alternatives, you would choose **d**. If given the choice between **a**, **b**, and **c**, you would choose **c**.

Exercise 8.12 Exponential discounting, cont. Suppose instead that $\delta = 0.1$.

- Compute the utility of each of the four utility streams from the point of view of $t = 0$.
- What would you choose if given the choice between all four?
- What if you had to choose between **a**, **b**, and **c**?

As these calculations show, your discount factor can have a dramatic impact on your choices. If your discount factor is high (that is, close to one) what happens in future periods matters a great deal. That is to say that you exhibit **patience**: you do not discount your future very much. If your discount factor

while, we let u_t denote the utility you receive today, and so on. We write $U^t(\mathbf{u})$ to denote the utility we seek is the utility

the *delta function*, the point of view of $t = 0$ is:

the utility you would receive the next round, δ^2 times as much, and so on. The resulting

as in Table 8.2. An empty set. In order to compute the utility at other points in time, you just need to be given the discount factor and the option you should choose.

counting

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4

4

Suppose that $\delta = 0.9$, and that each alternative has utility $u_0 = 1$, $U^0(\mathbf{b}) = \delta u_1 = 0.9$, $U^0(\mathbf{c}) = u_0 + \delta u_1 + \delta^2 u_2 = 1 + 2.7 \times 0.81 = 3.267$. When all four alternatives are available, you would choose c.

Suppose instead that $\delta = 0.1$. The utility streams from the point of view of $t = 0$ are:

between all four? Which one would you choose?

It can have a dramatic impact on your choice (that is, close to one) what happens in the future matters little. That is to say that you exhibit **impatience**: you discount your future heavily. It should be clear how the value of δ captures your time preference.

is low (that is, close to zero) what happens in the future matters little. That is to say that you exhibit **impatience**: you discount your future heavily. It should be clear how the value of δ captures your time preference.

Exercise 8.13 The ant and the grasshopper According to the fable, the grasshopper chirped and played all summer while the ant was collecting food. When winter came, the ant had plenty of food but the grasshopper died from hunger. What can you surmise about their deltas?

Economists believe discount factors can be used to explain a great deal of behavior. If your discount factor is low, you are more likely to spend money, procrastinate, do drugs, and have unsafe sex. If your discount factor is high, you are more likely to save money, plan for the future, say no to drugs, and use protection. Notice that this line of thought makes all these behaviors at least potentially rational. For somebody who discounts the future enough, there is nothing irrational about nurturing a crack-cocaine habit. In fact, Gary Becker (whom we came across in Section 2.8) is famous, among other things, for defending a theory of **rational addiction**.

Exercise 8.14 Discount factors For each of the following, identify whether the person's δ is likely to be high (as in close to one) or low (as in close to zero):

- A person who raids his trust fund to purchase a convertible.
- A person who enrolls in a MD/PhD program.
- A person who religiously applies sunscreen before leaving the house.

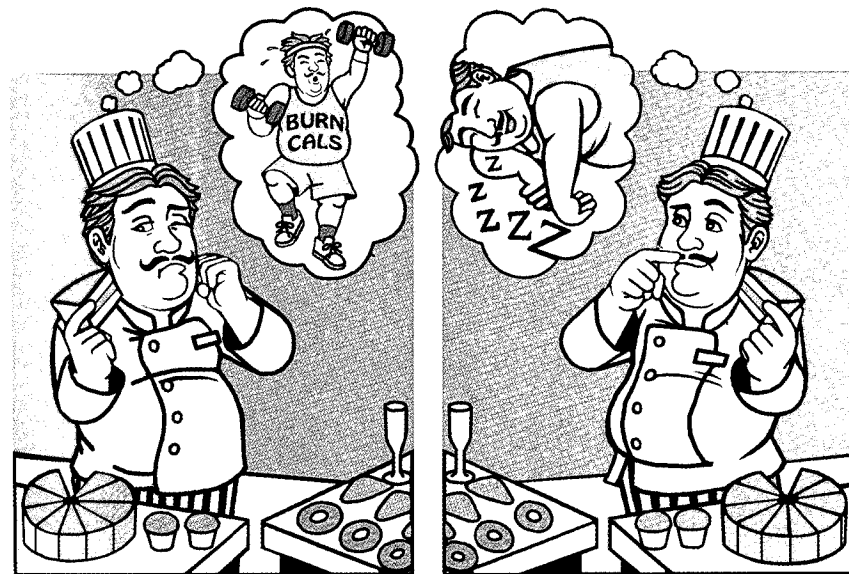


Figure 8.1 Time preference. Illustration by Cody Taylor

- (d) A person who skips her morning classes to go rock climbing.
 (e) The Iroquois Native American who requires that every deliberation must consider the impact on the seventh generation.

Exercise 8.15 The impartial spectator Adam Smith's *Theory of Moral Sentiments* made a big deal of the differences between an "impartial spectator" and our actual selves. An impartial spectator, Smith wrote, "does not feel the solicitations of our present appetites. To him the pleasure which we are to enjoy a week hence, or a year hence, is just as interesting as that which we are to enjoy this moment." When it comes to our actual selves, by contrast: "The pleasure which we are to enjoy ten years hence interests us so little in comparison with that which we may enjoy to-day." If we interpret the difference in terms of deltas, what would this entail when it comes to the deltas of (a) the impartial spectator, and (b) our actual selves?

Discounting can usefully be represented graphically. We put time on the x -axis and utility on the y -axis. A bar of height u at time t represents a reward worth u utiles to you when you get it at time t . A curve represents how much receiving the reward at t is worth to you from the point of view of times before t . As we know from Definition 8.10, that is δu at $t-1$, $\delta^2 u$ at $t-2$, and so on. As a result, we end up with a picture like Figure 8.2. As you move to the left from t , the reward becomes increasingly distant, and therefore becomes worth less and less to you in terms of utility.

We can use this graphical device to represent the difference between people with high and low δ s. If δ is high, δu will not differ much from u , and the curve will be relatively flat: it will only approach the x -axis slowly as you move to the left in the diagram, like the dashed curve in Figure 8.2. If δ is low, δu will be much lower than u , and the curve will be relatively steep: it will approach the x -axis quickly as you move to the left in the diagram, like the dot-dashed curve

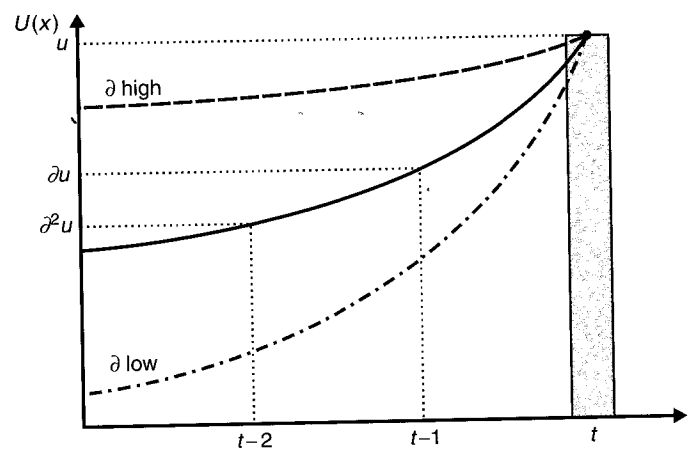


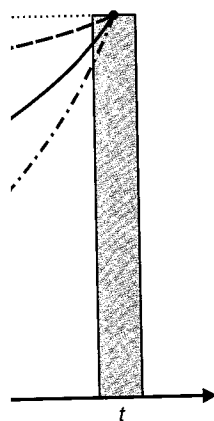
Figure 8.2 Exponential discounting

rock climbing.
 Every deliberation must

Smith's *Theory of Moral*
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curve in the figure. Again, if δ is high, the person does not discount the future very much, and the curve is flat; if δ is low, the person does discount the future a great deal, and the curve is steep.

So far, we have used our knowledge of δ to determine a person's preferences over utility streams. Definition 8.10 also permits us to go the other way, as it were. Knowing the person's preferences over utility streams, we can determine the value of her discount factor.

Example 8.16 Indifference Suppose that Alexandra, at time zero, is indifferent between utility streams **a** (2 utiles at $t = 0$) and **b** (6 utiles at $t = 1$). What is her discount factor δ ?

Given that Alexandra is indifferent between **a** and **b** at time zero, we know that $U^0(\mathbf{a}) = U^0(\mathbf{b})$, which implies that $2 = 6\delta$ which is to say that $\delta = 2/6 = 1/3$.

When experimental economists study time discounting in the laboratory, they rely heavily on this kind of calculation. As soon as a laboratory subject is indifferent between an immediate and a delayed reward, his or her discount factor can easily be estimated.

Exercise 8.17 A stitch in time "A stitch in time saves nine," people say when they want you to do something now rather than later. But not everyone will be swayed by that sort of concern. Suppose that you can choose between one stitch at time zero and nine stitches at time one, and that each stitch gives you a utility of -1 . What does it take for you to prefer the one stitch now?

Indifference can be represented graphically. Indifference between options **a** and **b** in Example 8.16 means that the picture would look like Figure 8.3. It can easily be ascertained that a strict preference for **a** over **b** would imply that $\delta < 1/3$ and that a strict preference for **b** over **a** would imply that $\delta > 1/3$.

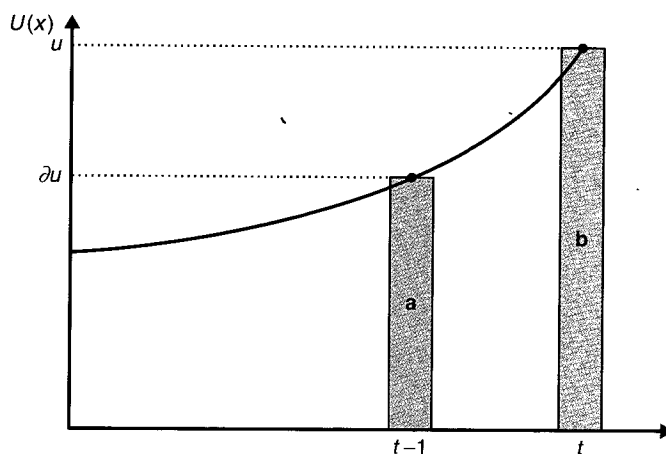


Figure 8.3 Exponential discounting with indifference

Exercise 8.18 Use Figure 8.3 to answer the following questions:

- (a) If $\delta < 1/3$, what would the curve look like?
 (b) What if $\delta > 1/3$?

Exercise 8.19 This exercise refers to the utility streams in Table 8.2. For each of the following people, compute δ .

- (a) At $t = 0$, Ahmed is indifferent between utility streams **a** and **b**.
 (b) At $t = 0$, Bella is indifferent between utility streams **b** and **c**.
 (c) At $t = 0$, Cathy is indifferent between utility streams **a** and **c**.
 (d) At $t = 1$, Darrence is indifferent between utility streams **b** and **c**.

Exercise 8.20 For each of the three decision problems in Table 8.3, compute δ on the assumption that a person is indifferent between **a** and **b** at time zero.

Table 8.3 Time-discounting problems

	$t = 0$	$t = 1$
a	3	4
b	5	1

(a)

	$t = 0$	$t = 1$	$t = 2$		$t = 0$	$t = 1$	$t = 2$
a		6	1	a	1		1
b		3	4	b			5

(b)

(c)

Exercise 8.21 As a financial advisor, you offer your clients the possibility to invest in an asset that generates a utility stream of 1 utile this year ($t = 0$), 0 utiles next year ($t = 1$), and 1 utile the year after that ($t = 2$). For each of the following clients, determine their δ .

- (a) Client P is indifferent between the investment and 2 utiles next year.
 (b) Client Q is indifferent between the investment and 1 utile this year.
 (c) Client R is indifferent between the investment and 1.25 utiles this year.

All the decision problems we have encountered so far in this chapter were defined by matrices of utilities. Very often, however, the relevant outcomes are given in dollars, lives saved, or the like. Given the appropriate utility function, however, we know how to deal with those problems too.

Example 8.22 Suppose you are indifferent between utility streams **a** and **b** in Table 8.4(a). Your utility function is $u(x) = \sqrt{x}$. What is your δ ?

Given the utility function, Table 8.4(a) can be converted into a matrix of utilities as in Table 8.4(b). We can compute δ by setting up the following equation: $3 + \delta 2 = 1 + \delta 5$. Or we can simply set up the following equation: $\sqrt{9} + \delta\sqrt{4} = \sqrt{1} + \delta\sqrt{25}$. Either way, the answer is $\delta = 2/3$.

Exercise 8.23 Suppose instead that the utility function is $u(x) = x^2$. What would Table 8.4(b) look like, and what would δ be?

Table 8.4 Time-discounting problem (in dollars and utiles)

	$t = 0$	$t = 1$		$t = 0$	$t = 1$
a	\$9	\$4	a	3	2
b	\$1	\$25	b	1	5

(a) In dollar terms

(b) In utility terms

Discount rates

Sometimes discounting is expressed in terms of a **discount rate** r rather than a discount factor δ . The conversion is easy:

$$r = \frac{1 - \delta}{\delta}$$

You can confirm that when $\delta = 1$, $r = 0$ and that as δ approaches zero, r increases. Knowing r , you can compute δ as follows:

$$\delta = \frac{1}{1 + r}$$

In this text, I favor discount factors over discount rates. But it is useful to know what both are.

8.4 What is the rational delta?

In the above, I have followed what is common practice and treated the value of the discount factor δ (and discount rate r) as a mere preference. On this approach, a person's discount factor is just a personal matter, much like your preference for blueberry ice-cream over raspberry ice-cream, or your preference for having ice-cream rather than throwing yourself out of the window. In other words, rationality does not require you to have one delta rather than another. One implication of this analysis, as we saw in the previous section, is that destructive behaviors like heavy drug use are perfectly consistent with rationality: if your discount factor is very low, aiming for instant gratification is perfectly rational in this analysis. Rationality does require you to consistently apply one discount factor, however. You cannot rationally plan for the future now and then act impulsively later; you cannot rationally act as though you have a high delta now and a low delta later. An inconsistent drug user is indeed irrational.

Historically, a number of thinkers have disagreed, and instead argued that time discounting (with $\delta < 1$ and $r > 0$) arises from some intellectual or moral deficiency. The economist A. C. Pigou, commonly considered the father of welfare economics, wrote that "this preference for present pleasures ... implies only that our telescopic faculty is defective, and that we,

ing questions:

reams in Table 8.2. For each

reams **a** and **b**.

reams **b** and **c**.

reams **a** and **c**.

reams **b** and **c**.

blems in Table 8.3, compute between **a** and **b** at time zero.

$t = 1$	$t = 2$
1	
5	

(c)

your clients the possibility of 1 utile this year ($t = 0$), or that ($t = 2$). For each of the

it and 2 utiles next year.

it and 1 utile this year.

it and 1.25 utiles this year.

id so far in this chapter were ver, the relevant outcomes are e appropriate utility function, lems too.

tween utility streams **a** and **b**. What is your δ ?

be converted into a matrix δ by setting up the following et up the following equation: r is $\delta = 2/3$.

y function is $u(x) = x^2$. What be?

therefore, see future pleasures, as it were, on a diminished scale." The polymath Frank P. Ramsey, who made path-breaking contributions to philosophy, statistics, and economics – his theory of saving is still taught in graduate-level macroeconomics courses – before dying at the age of 26, called time discounting "a practice which is ethically indefensible and arises merely from the weakness of the imagination." Some describe time discounting as a sort of crime committed by the current you against the future you – who is, they add, the person who should matter the most to you now. If these thinkers are right, repairing our intellectual and moral deficiencies would return us to a delta of one.

Others have advocated discounting the future. "Seize the day," or *carpe diem*, said the ancient poet Horace – and Robin Williams in *Dead Poets Society* – encouraging us to live in the present. Seneca, whom we came across in our discussion of the aspiration treadmill in Section 3.5, wrote:

Can anything be more thoughtless than the judgement of those men who boast of their forethought? ... They organize their thoughts with the distant future in mind; but the greatest waste of life consists in postponement: that is what takes away each day as it comes, that is what snatches away the present while promising something to follow. The greatest obstacle to living is expectation, which depends on tomorrow and wastes today.

This passage appears in a treatise titled *On the Shortness of Life*, in which Seneca argues that life is long enough – the problem being that so much of it is squandered. In the same spirit it is not uncommon for people to volunteer the advice that you should live every moment as though it were your last. The late Apple founder Steve Jobs was one of them: "I have looked in the mirror every morning and asked myself: 'If today were the last day of my life, would I want to do what I am about to do today?'" Then again, Harvard psychologist Daniel Gilbert quips that such a comment "only goes to show that some people would spend their final [moments] giving other people dumb advice." (Some of these figures might have been talking about uncertainty too, since the future is not only removed in time, it is also as yet unrealized.)

There is some evidence that having a high discount factor is good for a person in the long term. The 2014 book *The Marshmallow Test* reviews a series of studies performed with children as young as four, who are given the choice between one marshmallow (or similar treat) now, or two such treats if they are able to wait for some 15 minutes. Since the first experiment in the series was performed in the 1960s, experimenters have been able to track the performance of the original participants in a variety of domains of life. Remarkably, children who were able to defer gratification in preschool exhibited better concentration, intelligence, self-reliance, and confidence in adolescence and were better able to pursue and reach long-term goals, enjoyed higher educational attainment, and had a lower body-mass index in adulthood. This might strike you as an argument for having a high delta. Then again, if your delta is low now, it is not as though you will care about adverse long-term consequences of time discounting – or anything else.

8.5 Discussion

In this chapter we have explored issues of interest – simple and compound – as well as exponential discounting. The model of exponential discounting has a remarkable feature in that it is the only model of discounting that entails time consistency. Consistency is an important topic in intertemporal decision-making, and we will return to it in Section 9.2. Though relatively simple, exponential discounting offers an extraordinarily powerful model. For this reason, it is critical to a variety of fields, including in cost–benefit analysis, to assess the costs and benefits of deferred outcomes, and in finance, to determine the present value of alternative investments. While the model might seem intuitively appealing and uncontroversial from both a descriptive and a normative perspective, it is anything but.

In Section 8.4, we saw that a person’s discount factor δ is typically treated as a mere preference: as long as you are consistent in your attitudes to the future, rationality is consistent with any δ . In some contexts, however, we do not have the luxury of sidestepping the issue. When computing the costs and benefits of long-term investments, for example, some δ just has to be assumed and the specific value can matter hugely. Many investments (like railroads) have large upfront costs and benefits that extend into the indefinite future. If costs and benefits are computed with a low δ , the former will dominate and the project will be a no-go; if they are computed with a high δ , the latter will dominate, and the project will be a go. And there is simply no obvious, uncontroversial way to choose a number. Perhaps most prominently, the topic comes up in the discussion of climate change. On the assumption that taking action to prevent or mitigate the negative consequences of climate change would be costly in the short term but (at least potentially) beneficial in the long term, cost–benefit analysis with a high enough δ will favor taking action, but cost–benefit analysis with a low enough δ will favor not taking action. Thus, the rationality of taking action to prevent or mitigate the effects of climate change depends on the value of δ – and, again, there is no agreement whatsoever about the appropriate value of the parameter.

In the next chapter, we will discuss to what extent the exponential model of time discounting captures the manner in which people make decisions and whether it is appropriate as a normative standard.

ADDITIONAL EXERCISES

Exercise 8.24 Youth sports In a 2014 interview, basketball star Kobe Bryant discussed the importance of making sports fun for young people. “It’s hard to tell a kid that you need to get out there and compete because it’s going to decrease your chance of having diabetes 30–40 years from now,” he said. In the language of time discounting, why are young people not motivated by the prospect of being healthy in 30–40 years?

ADDITIONAL EXERCISES cont.

Exercise 8.25 Credit scores A study in the journal *Psychological Science* by two economists from the Federal Reserve Bank in Boston found a connection between people's discount factor δ and their credit score. A credit score is a number representing a person's creditworthiness: the higher the score the better. Based on your understanding of discounting behavior:

- What is the connection? That is, do people with higher discount factors tend to have higher credit scores, or the other way around?
- Explain why that would be so.

Exercise 8.26 Time discounting and interest rates Whether you should spend or save will depend not just on your time preference, but on the interest you can get when putting your money in a savings account. Suppose you have the option of spending $\$w$ now or saving it until next year. If you save it, the bank will pay you interest rate i .

- Suppose that your utility function is $u(x) = x$. What does your discount factor δ need to be for you to be indifferent between spending it now and saving it? What about your discount rate r (see the text box on page 191)?
- Suppose that your utility function is $u(x) = \sqrt{x}$. What does your discount factor δ need to be for you to be indifferent between spending it now and saving it? What about your discount rate r ?

FURTHER READING

Mas-Colell et al. (1995, Chapter 20) contains a more advanced discussion of intertemporal utility. The battle over payday loan establishments is covered by *the Wall Street Journal* (Anand, 2008). Smith (2002 [1759], p. 221–22) compares our actual selves to the impartial spectator. Pigou (1952 [1920], p. 25) discusses our flawed telescopic faculty and Ramsey (1928, p. 543) our weakness of imagination. Seneca (2007 [c 49], p. 148) and Jobs (2005) advocate living in the present, while Gilbert (2006, p. xiii) quips. *The Marshmallow Test* is Mischel (2014). Kobe Bryant is quoted in Shelburne (2014) and the study from the Boston Fed is Meier and Sprenger (2012).