Teorías del Crecimiento Económico
Lecture Notes on Structural Change

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Main Reference

Chapter 20 in Acemoglu, D., Introduction to Modern Economic Growth, Princeton University Press, 2009.

1 Introduction

The US economy has undergone important resource reallocations between sectors over the last two centuries. The importance of the agricultural sector has decreased steadily. That of the manufacturing sector increased unil the 1950's and it started falling after that. Services monotonically increased. The easiest way to show this is looking at the share of labor employed in each of these sectors.

Consumption shows similar trends although changes in relative prices keep agricultural consumption at relevant levels. Similar patterns can be found in most developed nations, and even developing nations exhibit signs of this process. Economist refer to this regularity as *Structural Change*, because the structure of the economies changes over time.

Structural Change is normally regarded as one of the main sources pushing economic growth. Caselli (2005) shows that even if sectoral productivity differences across countries are maintained, if the share of agricultural employment in every country was similar to that in the US, across country income inequality would be reduced by 2/3.

But what drives structural change? Intuitively, we can split reasons in two broad categories: demand and supply sided explanations. As we will see here, if sectors grow at different rates (supply-side explanation) it is possible to get a patterns as the one described above. Alternatively, if sectors grow at equal rates but consumer tastes for the output of different sectors changes over time (demand-side explanation), the value of output coming from different sectors can present trends as those above.

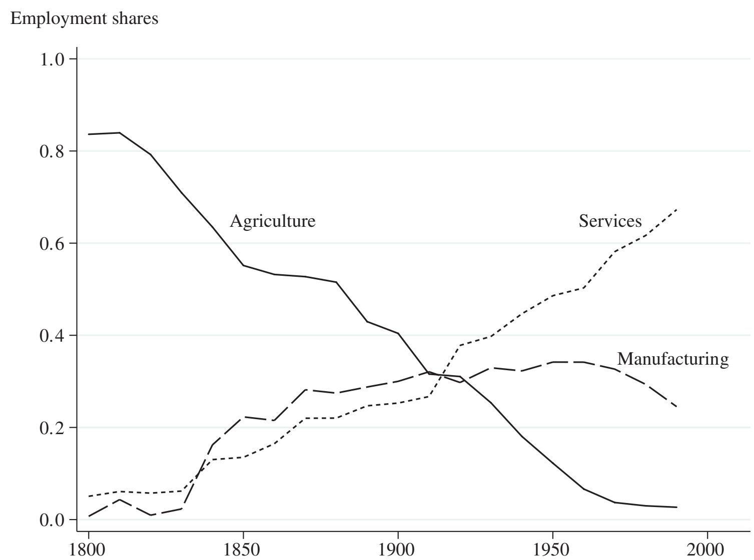

FIGURE 20.1 The share of U.S. employment in agriculture, manufacturing, and services, 1800–2000.

Another relevant question to ask is: how is structural change compatible with the Kaldor facts? In particular, can we have sectors changing their relative importance and, at the same time, factor shares and the interest rate being constant? In what follows we will cover both the supply and demand side explanations of structural change, paying particular attention at the conditions that need to hold for structural change to be compatible with the Kaldor facts.

Of course, in reality both supply and demand side effect are at play. Nevertheless, for the sake of transparency and tractability we will separate both types of effects.

2 Structural Change and the Kaldor Facts

2.1 The demand side

One of the main ingredients pushing Structural Change is what is known as the Engel's law (named after Ernst Engel). This *law* is actually a very robust empirical regularity: as households get richer, the share of expenditure they devote to food falls. The observation at the micro level is compatible with a setting in which food constitutes basic needs, while other goods satiate consumer's more sophisticated requirements. At the macro level, if we assume agricultural goods as the most basic and services as the most sophisticated, a pattern of structural change similar to that in the US can arise.

2.1.1 Kongsamut, Rebello and Xie (2001 REStud)

Kongsamut et al. (2001) refer to the inter-sectoral reallocation of resources as the *Kuznets facts*, since documenting this facts is credited to Simon Kuznetz (and also previously to C. Clark). These authors were the first providing a model to reconcile structural change with the Kaldor facts.

Consider an infinite-horizon economy where households supply labour inelastically and population grows at exogenous rate n, so aggregate supply at t is given by $L(t) = e^{nt}L(0)$. Intertemporal utility is given by:

$$U(t) = \int_0^\infty e^{-t(\rho-n)} \frac{c(t)^{1-\theta} - 1}{1-\theta} dt \tag{1}$$

where instantaneous utility is of the Constant Relative Risk Aversion type, with $\theta \geq 0$, and per capita consumption given by the Stone-Geary aggregate:

$$c(t) = (c^A(t) - \gamma^A)^{\eta^A} c^M(t)^{\eta^M} (c^S(t) + \gamma^S)^{\eta^S} \tag{2}$$

with c^i being the per capita consumption level of industry $i = A, M, S$, and $\sum_i \eta^i = 1$. Such preferences imply that households need to consume a *subsistence level* $\gamma^A > 0$ of the agricultural good (otherwise preferences are not defined). Similarly, the term $\gamma^S > 0$ makes consumers start spending money on S only after aggregate consumption surpasses that threshold. Together, both parameters guarantee that consumers first spend their budget on A, then on A and M, and spending on S starts only at a later stage, reproducing the pattern that we see in the data. Clearly, the function in (2) is not homogeneous of degree 1, which implies that preferences over consumption of different industries are *non-homothetic*.

Production functions take the form: $Y^i(t) = B^i F(K^i(t), X(t)L^i(t))$, for $i = A, M, S$. Here $F()$ satisfies the Inada conditions, and B^i and X are a Hick-Neutral and Harrod-neutral productivity terms respectively. Notice that production functions are the same for all three sectors. This forces reallocation between industries to come from the demand side in this model. Assume further that $L(0) > 0$, $K(0) > 0$ and $\dot{X}(t)/X(t) = g \forall t$ with $X(0) > 0$. The economy is closed, manufactures can be used as investment goods (K), and markets clear, which implies:

$$\begin{aligned} K^A(t) + K^M(t) + K^S(t) &= K(t) \\ L^A(t) + L^M(t) + L^S(t) &= L(t) \\ Y^A(t) &= c^A(t)L(t) \\ Y^S(t) &= c^S(t)L(t) \\ Y^M(t) &= c^M(t)L(t) + \dot{K}(t) \end{aligned}$$

All markets are competitive and the price of manufactures is the *numeraire*, which yields:

$$p^A(t) = \frac{c^M(t)}{c^A(t) - \gamma^A}\frac{\eta^A}{\eta^M} \qquad and \qquad p^S(t) = \frac{c^M(t)}{c^S(t) + \gamma^A}\frac{\eta^S}{\eta^M} \tag{3}$$

$$w(t) = \frac{B^M \partial F(K^M(t), X(t)L^M(t))}{\partial L^M}$$

$$r(t) = \frac{B^M \partial F(K^M(t), X(t)L^M(t))}{\partial K^M}$$

A competitive equilibrium is defined by

- a path of factor demands $[K^i(t), L^i(t)]_{t=0}^{\infty}$ maximizing profits given the path of aggregate supply $[K(t), L(t)]_{t=0}^{\infty}$ and a path of prices $[p^A(t), p^S(t), w(t), r(t)]_{t=0}^{\infty}$
- a path of prices $[p^A(t), p^S(t), w(t), r(t)]_{t=0}^{\infty}$ that clear markets given supplies and demands
- a path of consumption and savings $[c^i(t), K(t)]_{t=0}^{\infty}$ maximizing the intertemporal consumer problem.

Finally, assume that $B^A F(K^A(0), X(0)L^A(0)) > \gamma^A L(0)$, so the starting point of this economy covers the initial subsistence level of agricultural consumption. In any equilibrium, we have:

$$\frac{K^i(t)}{X(t)L^i(t)} = \frac{K(t)}{X(t)L(t)} = k(t) \quad \forall i = A, M, S$$

$$p^A(t) = \frac{B^M}{B^A} \qquad and \qquad p^S(t) = \frac{B^S}{B^A} \tag{4}$$

Notice that according to the previous expressions, prices are constant over time. Notice also that the previous result stems strictly from the technological side of the model (we did not use preferences for it).

Using preferences in (1) we get the following Euler equation:

$$\frac{\dot{c}(t)}{c(t)} = \frac{1}{\theta}[r(t) - \rho]$$

More importantly, combining (3) and (4) with can reach:

$$\frac{c^A(t) - \gamma^A}{c^M(t)} = \frac{B^A \eta^A}{B^M \eta^M} \quad and \quad \frac{c^S(t) + \gamma^S}{c^M(t)} = \frac{B^S \eta^S}{B^M \eta^M} \tag{5}$$

which highlights a key implication of constant prices: the ratio in the left hand side of each expression needs to be constant. Therefore, these expressions show how the rate of consumption growth is different in each industry as long as $\gamma^A, \gamma^S > 0$. In fact, it is easy to see that $\dot{c^A}/c^A < \dot{c^M}/c^M < \dot{c^S}/c^S$. This is the outcome of preferences that reproduce Engels' law, or put differently, preferences that are *non-homothetic*.

In a closed economy, differences in consumption growth across sectors necessarily translate into uneven production (and employment) growth between them. Despite sectors growing at uneven rates, we can prove that a unique equilibrium exists where c grows at a rate that is (asymptotically) constant.

Bottom line: when consumption in the different sectors evolve at uneven rates, a BGP with constant aggregate consumption is not possible. However, there exists a BGP where consumption grows at an **asymptotically** constant rate. $\gamma^A/B^A = \gamma^S/B^S$ is a necessary and sufficient condition for such BGP to exist.

In this equilibrium structural change takes place even though the interest rate and the share of capital in national income are constant. This model shows how structural change can be compatible with the Kaldor facts.

The model can be further expanded to make it even more realistic. The main shortcomings of the model are:

- some of the assumptions are too unrealistic but key for the results: same production function between sectors, only M being saved, etc.
- the necessary and sufficient condition for the BGP with constant consumption
  - is knife-edge and needs not holding.
  - violates independence between preferences and technology.

2.2 The supply side

If sectors grow at systematically different rates, one should expect the relative size of these sectors to change over time. This intuition was first formalized by Baumol (1967). In this chapter, we show how this can be consistent with the Kaldor facts. We follow Acemoglu and Guerrieri (2008) where uneven growth is the result of capital deepening in a context where there are sectoral differences in the proportion of factors usage. Another canonical explanation, based on exogenous TFP growth being uneven between sectors can be found in Ngai and Pissarides (2007).

Our first step is to present a general version of the model in Acemoglu and Guerrieri (2008) where functional specifications are avoided. We use this version of the model to show a simple result, i.e. that when sectors use factors at different proportions and one of these factors increases over time, then growth will not be balanced, even when technological progress is balanced. In a second step, we reduce the generality of the model introducing specific production and utility functions to gain tractability and we use that version of the model to show how models of unbalanced growth can be compatible with the Kaldor facts.

2.2.1 Acemoglu and Guerrieri (2008 JPE)-general version

The model consists of two intermediate sectors, each employing capital and labour to produce their output. These are then combined in the production of final output. All production functions satisfy Inada conditions and can be written as:

$$
\begin{aligned}
Y(t) &= F(Y_1(t), Y_2(t)) \\
Y_1(t) &= A_1(t)G_1(K_1(t), L_1(t)) \\
Y_2(t) &= A_2(t)G_2(K_2(t), L_2(t))
\end{aligned}
$$

where A_i represents a Hicks-neutral technology term in intermediate sector $i = 1, 2$. At a first stage we assume the path for A, K and L is given, which is useful to highlight that the processes leading technology to develop and factors to accumulate, are not important in this model. We will assume capital does not depreciate over time.

We take the final good as the *numeraire* in every period. We denote p_i the price of intermediate good $i = 1, 2$. All markets clear so $K_1(t) + K_2(t) = K(t)$ and $L_1(t) + L_2(t) = L(t)$ and:

$$
\frac{p_1(t)}{p_2(t)} = \frac{\partial F/\partial Y_1}{\partial F/\partial Y_2} \tag{6}
$$

$$
w(t) = p_1(t)A_1(t)\frac{\partial G_1}{\partial L_1} = p_2(t)A_2(t)\frac{\partial G_2}{\partial L_2} \tag{7}
$$

$$
r(t) = p_1(t)A_1(t)\frac{\partial G_1}{\partial K_1} = p_2(t)A_2(t)\frac{\partial G_2}{\partial K_2}
$$

Let us define:

- equilibrium: given a supply path for factors $K(t)$ and $L(t)$, an equilibrium is a path of product and factor prices and allocations $[p_i(t), w(t), r(t), K_i(t), L_i(t)]_{t=0}^{\infty}$ such that all markets clear and all agents find a solution to their optimization problems.
- capital shares in sector i: $\sigma_i(t) = r(t)K_i(t)/[p_i(t)Y_i(t)]$
- factor proportion differences: situation in which $\sigma_1(t) \neq \sigma_2(t)$
- capital deepening: situation in which $\dot{K}(t)/K(t) > \dot{L}(t)/L(t)$
- balanced technological progress: situation in which $\dot{A}_1(t)/A_1(t) = \dot{A}_2(t)/A_2(t)$
- unbalanced growth: situation in which $\dot{Y}_1(t)/Y_1(t) \neq \dot{Y}_2(t)/Y_2(t)$

Using these definitions it is possible to show that if at some t there are factor proportion differences between the two sectors and there is capital deepening, then growth is not balanced, even when technological progress is balanced.

The intuition for this result is straightforward. Suppose that there is capital deepening and sector 2 is more capital intensive. If both capital and labour were allocated to the two sectors at constant proportions over time, the more capital-intensive sector, would grow faster.

2.2.2 Acemoglu and Guerrieri (2008 JPE)-particular version

The structure of the model is similar to the one in the previous section. Time in continuous and infinite. Population grows at rate $n > 0$. A representative household supplies labour inelastically and has preferences given by (1).

The general production function now takes the CES form:

$$Y(t) = \left[\gamma Y_1(t)^{\frac{\epsilon-1}{\epsilon}} + (1-\gamma) Y_2(t)^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}} \tag{8}$$

where $\epsilon > 0$ represents the elasticity of substitution between the two intermediate goods and $\gamma \in (0,1)$ is a shifter determining the importance of each intermediate in final good production.

There is no depreciation and the final good can be consumed or saved so we can write the aggregate resource constraint of this economy as:

$$\dot{K}(t) + L(t)c(t) = Y(t) \tag{9}$$

Production of intermediates follow symmetric Cobb-Douglas specifications:

$$Y_i(t) = A_i(t) K_i(t)^{\alpha_i} L_i(t)^{1-\alpha_i}$$

Here α_i governs capital intensity in sector $i = 1, 2$. We assume $0 < \alpha_1 < \alpha_2 < 1$ which implies that sector 1 is less capital intensive than sector 2. $A_i(t)$ is the Hick-neutral technology factor. We assume that this term grows at an exogenous positive rate at every moment in time t, i.e.: $\dot{A}_i(t)/A_i(t) = a_i > 0 \forall t$. Notice that in this version of the model, capital deepening is now the outcome of growth, which in itself stems from exogenous technological progress.

Markets clear so we have $L_1(t) + L_2(t) = L(t)$ and $K_1(t) + K_2(t) = K(t)$. Again we set the price of the final good as the *numeraire*. Profit maximization in the final good sector gives:

$$p_1(t) = \gamma \left(\frac{Y_1(t)}{Y(t)}\right)^{-1/\epsilon} \quad and \quad p_2(t) = (1-\gamma)\left(\frac{Y_2(t)}{Y(t)}\right)^{-1/\epsilon} \tag{10}$$

Notice that (10) implies that:

$$\frac{p_1(t)}{p_2(t)} = \frac{\gamma}{1-\gamma}\left[\frac{Y_2}{Y_1}\right]^{1/\epsilon}$$

The above expression describes a positive relationship between the ratio p_1/p_2 and Y_2/Y_1 given the plausible values of the parameters. This shows that if output grows in an unbalanced fashion (i.e. Y_2/Y_1 is not constant), relative prices will move to offset changes in relative quantities. The strength of such movement depends on ϵ. If $\epsilon > 1$ (intermediate goods are gross substitutes), the change in relative output is only partially offset by the change in relative prices. If $\epsilon < 1$ (intermediate goods are gross complements), the change in relative output is more than offset by the change in relative prices, so the sector in which output grows less actually expands in value. Empirical work[1] has shown $\epsilon < 1$ to be the relevant case, so our focus will be placed in this scenario.

Firms' optimization gives:

$$
\begin{aligned}
w(t) &= \gamma(1-\alpha_1)\left(\frac{Y(t)}{Y_1(t)}\right)^{1/\epsilon}\frac{Y_1(t)}{L_1(t)} = (1-\gamma)(1-\alpha_2)\left(\frac{Y(t)}{Y_2(t)}\right)^{1/\epsilon}\frac{Y_2(t)}{L_2(t)} \\
r(t) &= \gamma\alpha_1\left(\frac{Y(t)}{Y_1(t)}\right)^{1/\epsilon}\frac{Y_1(t)}{K_1(t)} = (1-\gamma)\alpha_2\left(\frac{Y(t)}{Y_2(t)}\right)^{1/\epsilon}\frac{Y_2(t)}{K_2(t)}
\end{aligned}
$$

Let us define the fraction of capital and labour employed in sector 1 as $\kappa(t) = K_1(t)/K(t)$ and $\lambda(t) = L_1(t)/L(t)$ respectively. Combining previous results we obtain:

$$
\begin{aligned}
\kappa(t) &= \left[1+\frac{\alpha_2}{\alpha_1}\left(\frac{1-\gamma}{\gamma}\right)\left(\frac{Y_1(t)}{Y_2(t)}\right)^{\frac{1-\epsilon}{\epsilon}}\right]^{-1} \qquad (11)\\
\lambda(t) &= \left[1+\frac{\alpha_1}{\alpha_2}\left(\frac{1-\alpha_2}{1-\alpha_1}\right)\left(\frac{1-\kappa(t)}{\kappa(t)}\right)\right]^{-1}
\end{aligned}
$$

Notice that the share of labour in sector 1, is monotonically increasing in the share of capital in that sector. This means that in equilibrium if capital is growing in a given sector, labour will be growing in that sector too, so one sector will be expanding its usage of productive factors, while the other sector will be reducing that use.

How does the allocation of factors depend on the aggregate supply of each factor? We can show that

$$
\begin{aligned}
\frac{dlog\kappa(t)}{dlogK(t)} = -\frac{dlog\kappa(t)}{dlogL(t)} &= \frac{(1-\epsilon)(\alpha_2-\alpha_1)(1-\kappa(t))}{1+(1-\epsilon)(\alpha_2-\alpha_1)(\kappa(t)-\lambda(t))} \\
\frac{dlog\kappa(t)}{dlogA_2(t)} = -\frac{dlog\kappa(t)}{dlogA_1(t)} &= \frac{(1-\epsilon)(1-\kappa(t))}{1+(1-\epsilon)(\alpha_2-\alpha_1)(\kappa(t)-\lambda(t))}
\end{aligned}
$$

It is clear that the first expression is positive if and only if $(1-\epsilon)(\alpha_2-\alpha_1) > 0$, while the second expression is positive if and only if $\epsilon < 1$. According to the first expression, when $\epsilon < 1$ the fraction of capital allocated to the capital-intensive sector falls as the stock of capital increases. This is because when both intermediate goods are gross complements and capital increases, output in the capital intensive sector will grow more, but the produced value will fall relative to the other sector. This pushes a greater share of capital to be allocated in the less capital-intensive sector 1. A similar logic explains why, according to the second expression, when $\epsilon < 1$ improvement in the technology of a sector causes the share of capital going to that sector to fall. Alternatively, when $\epsilon > 1$ the exact opposite is true.

At this point we have a very good idea of how the mechanics of the model work in its static equilibrium, and which are the key parameters affecting those

[1] See for example Herrendorf et al. (2013).

mechanics. Now, what does the model say about unbalanced growth and the Kaldor facts? To answer this, it is crucial to understand how factors of production complement each other in the production of intermediates and what impacts the accumulation of each factor.

Defining capital share of the economy as before:

$$\sigma_K(t) = \frac{r(t)K(t)}{Y(t)} = \frac{\gamma\alpha_1}{\kappa(t)}\left(\frac{Y_1(t)}{Y(t)}\right)^{\frac{\epsilon-1}{\epsilon}}$$

Then we can show that:

$$\frac{dlog\sigma_K(t)}{dlogK(t)} < 0 \iff \epsilon < 1$$

A negative relationship between the share of capital in national income and the stock of capital necessarily means that K and L are gross complements in the aggregate production of the economy. The above result shows that the elasticity of substitution between capital and labour is less than 1 if and only if the elasticity of substitution between sectors' output is less than one ($\epsilon < 1$).

Finally, we are ready to move towards the analysis of the dynamic equilibrium of this model. A dynamic equilibrium is given by factor prices and allocations $[w(t), r(t), \kappa(t), \lambda(t)]_{t=0}^{\infty}$ satisfying the static equilibrium conditions at each t, and paths of aggregates $[c(t), K(t), Y(t)]_{t=0}^{\infty}$ satisfying the dynamic conditions of the model. Maximization of (1) gives the typical Euler equation for the path of consumption: $\frac{\dot{c}(t)}{c(t)} = \frac{1}{\theta}[r(t)-\rho]$. The remaining equations characterizing the dynamic equilibrium are a trasversality condition and the resource constraint in (9).

Let us define $\dot{L}_i(t)/L_i(t) = n_i(t)$, $\dot{K}_i(t)/K_i(t) = z_i(t)$, $\dot{Y}_i(t)/Y_i(t) = g_i(t)$ $\forall i = 1, 2$, and $\dot{K}(t)/K(t) = z(t)$ and $\dot{Y}(t)/Y(t) = g(t)$. Let us also define the following asymptotic growth rates (when they exist): $n_i^* = \lim_{t\to\infty} n_i(t)$, $z_i^* = \lim_{t\to\infty} z_i(t)$ and $g_i^* = \lim_{t\to\infty} g_i(t)$. Similarly, asymptotic factor allocations are defined as: $\kappa^* = \lim_{t\to\infty}\kappa(t)$ and $\lambda^* = \lim_{t\to\infty}\lambda(t)$.

Then, the following result can be shown:

$$\text{if } \epsilon < 1, \text{ then} \quad n_1(t) \gtreqless n_2(t) \Leftrightarrow z_1(t) \gtreqless z_2(t) \Leftrightarrow g_2(t) \gtreqless g_1(t)$$
$$\text{if } \epsilon > 1, \text{ then} \quad n_1(t) \gtreqless n_2(t) \Leftrightarrow z_1(t) \gtreqless z_2(t) \Leftrightarrow g_1(t) \gtreqless g_2(t)$$

The previous result highlights that when sectors are gross complements, the equilibrium growth rate of the capital stock and labour force in the sector that is growing faster must be less than in the other sector. The intuition for this is similar to what has been explained before: when the elasticity of substitution between the two sectors is less than 1, changes in relative prices more than offset changes in relative quantities so the value of the dynamic sector falls and that of the lagging sector grows. Such effect provides incentives for resources to be increasingly allocated to the lagging sector.

Another important result, it is possible to show that if asymptotic rates g_1^* and g_2^* exists, then: when $\epsilon < 1$ then $g^* = min\{g_1^*, g_2^*\}$, and conversely when $\epsilon > 1$ then $g^* = max\{g_1^*, g_2^*\}$.

The intuition behind this result is straightforward and directly obtains from the preceding result. The asymptotic growth rate of the economy will approach that of the sector growing in size. If resources are increasingly allocated to the sector that grows slower when $\epsilon < 1$ (faster when $\epsilon > 1$), then the long-term growth rate of the aggregate economy will be the lower (higher) of the two sectoral growth rates.

Let us focus in the asymptotic equilibrium in which per capita consumption grows at constant rate (g_c^*), which implies that r is constant and aggregate consumption grows at constant rate ($g_C^* = g_c^* + n$). For the final result let us assume that either $a_1/(1-\alpha_1) < a_2/(1-\alpha_2)$ and $\epsilon < 1$, or alternatively $a_1/(1-\alpha_1) > a_2/(1-\alpha_2)$ and $\epsilon > 1$. This assumption ensures that sector 1 is the asymptotically dominant sector, either because it has a slower (augmented) rate of technological progress and $\epsilon < 1$, or because it has more rapid (augmented) technological progress and $\epsilon > 1$.[2] Under this setting there exists a unique BGP with asymptotically constant consumption and is characterized by:

$$
\begin{aligned}
g^* &= g_C^* = g_1^* = z_1^* = n + g_c^* = n + \frac{a_1}{1-\alpha_1} \\
z_2^* &= n - (1-\epsilon)a_2 + (1 + (1-\epsilon)(1-\alpha_2))\frac{a_1}{1-\alpha_1} < g^* \\
g_2^* &= n + \epsilon a_2 + (1 - \epsilon(1-\alpha_2))\frac{a_1}{1-\alpha_1} > g^* \\
n_1^* = n, \text{ and } n_2^* &= n - (1-\epsilon)(1-\alpha_2)\left(\frac{a_2}{1-\alpha_2} - \frac{a_1}{1-\alpha_1}\right) < n_1^*
\end{aligned}
$$

It is important to highlight the following consequences of this result:

1. If $a_1/(1-\alpha_1) \neq a_2/(1-\alpha_2)$ growth is uneven between sectors. This is the result of combining uneven factor intensities with capital deepening: output growth will be faster in the more capital-intensive sector.

2. At equilibrium $\lambda^* = \kappa^* = 1$, so sector 1 converges towards a situation of full usage of both productive factors, while sector 2 shrinks. Nevertheless, at all points in time both sectors produce positive amounts. Moreover, the sector that is shrinking in terms of capital and labour share grows faster than the rest of the economy at all points in time.

3. The more slowly growing sector determines the long-run growth rate of the economy, while the more rapidly growing sector continually sheds factors but does so at exactly the rate to ensure that it still grows faster than the rest of the economy. In fact, the rate at which capital and labour are allocated away from this sector is determined in equilibrium to be exactly such that this sector still grows faster than the rest of the economy.

[2] Remember that in this model, besides technological progress, sectors also experience endogenous capital deepening. The overall effect on labour productivity (and output growth) depends on the rate of technological progress augmented with the rate of capital deepening, so the relevant rate to consider in sector $i = 1, 2$ is $a_i/(1-\alpha_i)$.

4. At equilibrium, the capital share in national income and the interest rate are constant. The asymptotic capital share in national income always reflects the capital share of the (asymptotically) dominant sector.

This model based on technological sources of unbalanced growth replicates the Kuznets facts while, at the same time proves compatible with the Kaldor facts.

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