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Game Theory in Economics and Beyond

Larry Samuelson

Within economics, game theory occupied a rather isolated niche in the 1960s and 1970s. It was pursued by people who were known specifically as game theorists and who did almost nothing but game theory, while other economists had little idea what game theory was. Game theory was taught only in occasional specialty courses. Nonetheless, game theory was surrounded by a buzz of anticipation and excitement, especially moving into the 1980s and early 1990s.

Game theory is now a standard tool in economics. Contributions to game theory are made by economists across the spectrum of fields and interests, and economists regularly combine work in game theory with work in other areas. Students learn the basic techniques of game theory in the first-year graduate theory core. Excitement over game theory in economics has given way to an easy familiarity.

This essay examines this transition, arguing that the initial excitement surrounding game theory has dissipated not because game theory has retreated from its initial bridgehead, but because it has extended its reach throughout economics. We begin with an overview of the development of game theory, with emphasis on its integration with economics. In the process, both the practice of economics and the nature of game theory have been transformed. We then turn to some key challenges for game theory, including the continuing problem of dealing with multiple equilibria, the need to make game theory useful in applications, and the need to better integrate noncooperative and cooperative game theory. The paper concludes with brief remarks about the current status and future prospects of game theory.

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Game Theory in Economics

Aggregating Individual Behavior

The social sciences are distinguished from one another not so much by what they study, but by how they study their subject. Economists stand out among the social sciences for their belief in *methodological individualism*—the tenet that explanations of social phenomena should be built up from the study of individual behavior—and the further belief that, within this paradigm, a common and parsimonious model can be consistently applied to examine whatever question arises. This model is built up from two principles. The first principle addresses individual behavior. The assumption here is that people have consistent and stable preferences, and that they choose the alternative from the set of feasible alternatives that is ranked highest under these preferences. The second principle addresses the aggregation of individual behavior to examine more complex phenomena. The standard organizing principle here was once the concept of a competitive market, with occurrences of market power viewed as exceptional cases. These twin principles were evident in the standard first-year graduate theory sequence, which consisted of a semester studying theories of optimization and its application to consumer and firm behavior, followed by a semester studying competitive equilibrium.

As the theory of competitive markets was reaching its culmination in Arrow and Debreu (1954), Debreu (1959), and McKenzie (1954) (see Düppe and Weintraub 2014 for a historical account), the foundations of game theory were also being laid (von Neumann and Morgenstern 1944; Nash 1950a, b, 1951, 1953). Game theory retains the familiar model of individual behavior, but offers an alternative and more general view—containing competitive markets as a limiting case—of how models of individual behavior are aggregated to examine more complex phenomena. Game theory has subsequently become the standard organizing principle for examining interactions between people, and has become established as the second pillar of methodological individualism. One again sees this evolution in the typical first-year graduate theory sequence, where general equilibrium theory has been nearly swept off stage in order to make room for game theory.

It will help to maintain a running example. We begin with the simplest model of a Cournot (1838) duopoly. Firms 1 and 2 simultaneously choose quantities of a homogenous good that they costlessly produce and sell. They sell their outputs at a common price, determined by a linear market demand function that gives the price as a function of the total quantity produced by the two firms. Figure 1 illustrates the Nash equilibrium¹ of this Cournot duopoly game. The point of departure is a market demand function, presumably derived from utility-maximization models describing the price-taking consumers in the market. A model based on competition would similarly derive a supply curve from profit maximizing models of price-taking firms. The Cournot model instead assumes there is a small number of firms (for

¹ A Nash equilibrium is a profile of strategies, one for each player, with the property that each player's strategy maximizes that player's payoff, conditional on the strategies of the other players.

simplicity of illustration, two), each cognizant of its effect on price, and seeks quantities satisfying the equilibrium condition that each firm maximizes profits given the residual demand function induced by the other firm's quantity. One can view a competitive market as the limiting case of this model as the number of firms grows arbitrarily large.

Classical Game Theory

Game theory has been transformed as it has percolated into economics. Game theory was initially dominated by a classical view, whose key component was that the game should be viewed as a literal description of the situation of interest, rather than just an approximation. Perhaps the clearest statement of this classical view appears in Kohlberg and Mertens (1986, p. 1005), who state:

We adhere to the classical point of view that the game under consideration fully describes the real situation—that any (pre)commitment possibilities, any repetitive aspect, any probabilities of error, or any possibility of jointly observing some random event, have already been modelled in the game tree. ... In principle, in situations where those restrictions are not met, the game tree is just used as a shorthand notation for the rules of a much bigger 'extended game' ... , and it is the stability of the equilibria of the extended game that has to be analyzed.

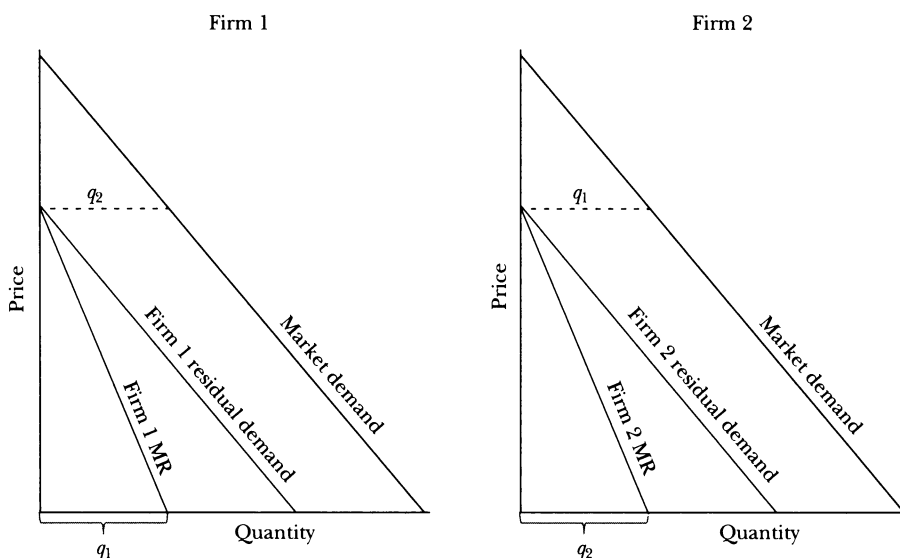
The classical view makes game theory neatly self-contained. There is no need to worry about whether the players in the game can communicate, or make agreements, or collude, or send signals to one another, or make commitments, and so on. If any of these were possible, they would be already included as moves in the game.

For example, Cournot's (1838) model of imperfect competition viewed firms as choosing their quantities of output, and then selling these outputs at a common market price determined by the total quantity produced in the market, as in Figure 1. In 1883, with the dates perhaps reflecting a slower pace of academic life, Bertrand wrote a review of Cournot's work (from 1838), arguing that firms should be modeled as choosing prices rather than quantities of output.² When the firms set different prices, all consumers buy from the lower-priced firm. The differing implications are dramatic. In the market portrayed in Figure 1, Cournot's firms choose quantities that lead to a market price higher than marginal cost and to positive profits, while ruthless price-cutting forces Bertrand's firms to set prices equal to marginal cost and to earn zero profits. How do we choose between the two models? Under the classical view, the answer is conceptually straightforward—we should check what firms actually do. If they choose quantities, we should use the Cournot model. If they set prices, we should use the Bertrand model. If they do

²For translations of Bertrand (1883) and the relevant chapter of Cournot (1838), see Daugherty (1988).

Figure 1

Illustration of Nash Equilibrium of Cournot Duopoly



Note: Firm 1 faces a residual demand function, given by the market demand function shifted to the left by the quantity q_2 of Firm 2's output, and maximizes profits by choosing that value q_1 that sets the resulting marginal revenue equal to the marginal cost, zero in this case because production is assumed to be costless. Firm 2 does likewise. Each firm's equilibrium quantity of output is a best response (that is, it maximizes profits) given the quantity produced by the other firm (that is, given the residual demand function induced by the other firm's quantity).

some combination of the two, or do something else, then we need a different model (for example, Kreps and Scheinkman 1983).

Once one has selected the appropriate game, attention typically turns to equilibrium behavior. Under the classical view of game theory, one should be able to deduce the equilibrium play from the specification of the game and the hypothesis that it is commonly known that the players are rational. An analyst observing the game should be able to make such a deduction, as should the players in the game. This immediately answers an obvious question: Why are we interested in the equilibrium of a game? In the classical view, the equilibrium implication of a game will be obvious to rational players, and will just as obviously be reflected in their behavior.

In the Cournot duopoly of Figure 1, it is straightforward to identify Nash equilibrium behavior, and to ascertain that there is only one such equilibrium. In general, however, games have many equilibria. Suppose, for example, that our two firms from Figure 1 interact not just once, but repeatedly. It is an equilibrium for the firms to act in each period just as they do in the equilibrium of the one-shot game. However, following the lead of Friedman (1971), if the firms are sufficiently patient, it is also an equilibrium for them to set the monopoly price and share the

monopoly profits in each period, with any cheating on such collusion prompting a switch to the behavior described in the preceding sentence. Indeed, the folk theorem (Fudenberg and Maskin 1986) tells us that with sufficiently patient players, virtually anything is an equilibrium outcome.

Multiplicity of equilibria is not limited to repeated games. If the firms in Figure 1 faced nonlinear demand functions or nontrivial cost functions, there could well be multiple equilibria. Settings characterized by uncertainty, such as signaling models, are well-known breeding sites for multiple equilibria. More generally, multiple equilibria arise in many settings for many reasons. How are we to identify the equilibrium implication of the game in the presence of multiple equilibria?

Equilibrium Refinements

The response to this question was the equilibrium refinements literature (van Damme 1991, 1992), which sought “refinement” criteria for limiting attention to a subset of the set of Nash equilibria. For example, one might restrict attention to Nash equilibria that do not play weakly dominated strategies.³ For much of the 1980s, work on refinements lay at the center of game theory and economic theory more generally. The holy grail of this quest was an equilibrium notion that economists and game theorists could embrace as *the* equilibrium notion, giving rise to a unique specification of play in any game to which it might be applied. Perhaps the culmination of the refinements program was Harsanyi and Selten’s (1988) theory of equilibrium selection, which indeed delivered unique outcomes, but is now most often cited for having introduced the distinction between risk dominance and payoff dominance.⁴

The equilibrium refinements literature was not a complete success. Instead of producing an equilibrium refinement that could command consensus, the literature gave rise to an ever-growing menagerie of refinements. New refinements tended to give rise to examples highlighting their weaknesses, followed by successor refinements destined to serve as the raw material for the next round. This seemingly endless cycle prompted Binmore (1992, p. 1) to liken the refinements quest

³A strategy for a player is weakly dominated if there exists an alternative strategy that ensures the player a payoff that is always (that is, for any opponents’ strategy) at least as high as that of the dominated strategy, and that is sometimes (that is, for some opponents’ strategy) strictly higher.

⁴In two-by-two symmetric games with two symmetric pure equilibria, one of the equilibria payoffs dominates the other if the former gives both players higher payoffs. One equilibrium risk dominates the other if the former involves strategies that are strict best responses to equal mixtures on the part of opponents, allowing an interpretation of being less subject to the risk that the opponent may not play her equilibrium strategy. Harsanyi and Selten (1988) has become a contender for being the most frequently misquoted work in game theory. It is common to find assertions that Harsanyi and Selten’s theory of equilibrium selection selects the risk-dominant equilibrium in two-by-two coordination games. Contrast this with their statements: “The solution function ... for the application of our general concept to this class [2x2 games with two strict Nash equilibria] gives absolute priority to payoff dominance” (p. 90) and “Risk dominance and payoff are combined to form a *dominance relationship* that gives precedence to payoff dominance” (p. 196).

to Hercules' quest to kill the Hydra, with two new heads appearing in the place of each predecessor.

At the same time that many game theorists were busy inventing and reinventing refinements of Nash equilibrium, difficulties appeared in the attempt to show that at least Nash equilibrium, much less refinements of Nash equilibrium, could be deduced from the specification of the game and the hypotheses that the players are commonly known to be rational. The opening salvos in this investigation, by Bernheim (1984) and Pearce (1984), concluded that the common knowledge of rationality allowed one to infer only that players will restrict attention to rationalizable strategies. In some games, this is enough. To identify rationalizable strategies in the Cournot duopoly game of Figure 1, for example, one first observes that no firm would ever find it optimal to produce more than the monopoly quantity, no matter what it thinks the other firm will do. One can then eliminate any quantity larger than the monopoly quantity as not being rationalizable, putting an upper bound on the set of rationalizable strategies. Next, if a firm is confident its opponent will not produce more than the monopoly quantity, then the firm will never want to produce less than the optimal output corresponding to the residual demand function created by the opponent's production of the monopoly quantity. We can thus eliminate lower quantities, putting a lower bound on the set of rationalizable strategies. But now, knowing that no firm would produce less than this lower bound, we can deduce that no firm would find it optimal to produce the entire monopoly output. This tightens the upper bound on the set of rationalizable strategies. It is not immediately obvious, but it is straightforward to show that in the case of the Cournot duopoly shown in Figure 1, this process continues until there is only one rationalizable strategy left standing, which is the Nash equilibrium output. Here, the common knowledge of rationality very neatly gives rise to Nash equilibrium.

Unfortunately, many other cases do not work nearly as well. Consider the matching pennies game. In one version of this strategic problem, Sherlock Holmes and James Moriarty (in a scene from Sir Arthur Conan Doyle's "The Final Problem") are aboard separate east-bound trains from London, each with the option of alighting at either Canterbury or Dover. Moriarty wins (and Holmes loses, with payoffs 1 for Moriarty and -1 for Holmes) if they choose the same stop, whichever it is, and Holmes wins (and Moriarty loses) if they choose different stops. This game has a unique Nash equilibrium, in which each player chooses each strategy with probability one-half, perhaps achieved by tossing a coin and choosing Canterbury if heads, Dover if tails. In contrast to this unique Nash equilibrium, every strategy is rationalizable. In particular, every option available to Holmes, including alighting at Canterbury, alighting at Dover, and any random choice between the two, is a best response to something Moriarty might do (and vice versa for Moriarty). In this setting, the rationalizability calculation thus never eliminates any strategies. This example is not an isolated one—rationalizability often has little bite.

The refinements literature has faced challenges on two fronts. Arguments based on a formal examination of the common knowledge of rationality prompted people to argue that sometimes even the Nash equilibrium notion was too restrictive, while

those in pursuit of refinements argued that sometimes the Nash equilibrium notion was too permissive.

An Instrumental View of Game Theory

In response, the classical view of game theory gave way to an instrumental view. In this view, the game is not a literal description of an interaction, but is a model that one hopes is useful in studying that interaction. In the words of Aumann (2000, p. 38; originally 1985), “Game-theoretic solution concepts should be understood in terms of their applications, and should be judged by the quantity and quality of their applications.” The game is thus a deliberate approximation, designed to include important aspects of the interaction and exclude unimportant ones. Under this view, for example, the choice between the Cournot and Bertrand models hinges not on what one thinks firms actually do (though talking to people who run firms might be a good source of intuition and inspiration), but on which model gives the most useful insights. Are we working in a setting in which competition between even two firms is enough to drive prices to marginal cost? If so, the Bertrand model may be appropriate. Do we think that the entry of a new firm into the market is likely to decrease the profits of existing firms? If so, the Cournot model is likely to be appropriate.

An implication of the instrumental view is that making a model more realistic does not necessarily make it a better model. It is obvious that making a model more *complicated* does not necessarily make it a better model. After all, as Lewis Carroll (1893, p. 169) wonderfully illustrated with the vision of a map on the scale of one-to-one, a model as complicated as its intended application is also typically useless. The more important point is that, even without extra complication, more realism need not be a step forward for a model. For example, models of infinitely repeated games are often criticized because “nobody lives forever.” A more realistic model would incorporate a finite horizon.⁵ However, the relevant considerations when assessing the time horizon revolve around human behavior rather than human mortality tables (for a discussion, see Osborne and Rubinstein 1994, p. 135). Do people allow end-game considerations to affect their behavior in the early periods of a repeated interaction? For example, suppose an antitrust case hinged upon the accusation that two firms in a repeated version of the market captured in Figure 1 were colluding by continually jointly producing the monopoly output, sustaining this behavior by the realization that any deviation would prompt a switch to the perpetual play of the less-lucrative Nash equilibrium of the one-shot game. This behavior is an equilibrium in an infinitely-repeated interaction between the two firms (given sufficient patience), but not in a finitely-repeated interaction with a fixed end period. In the latter case, the candidate monopoly equilibrium unravels

⁵ Arguments that we can think of the repeated game as a game whose length is finite but uncertain do not address this concern, since they give analogous results only if the continuation probability is bounded sufficiently far away from zero, which is no more realistic than an infinite lifetime. See Mailath and Samuelson (2006, chap. 4) for a discussion.

on the strength of a backward-induction argument. In particular, producing the joint monopoly output is not an equilibrium in the final period, where the only equilibrium behavior is given by the Nash equilibrium described in Figure 1. But once we have locked down behavior in the final period, the Nash equilibrium described in Figure 1 is the only equilibrium behavior in the penultimate period (since no variations in continuation play are available to induce players to take any other action), and similarly in all previous periods. However, suppose that the defense team in the antitrust case argued that the life of our planet is surely finite (since the sun will expire in a few billion years), making this a finitely repeated game and hence ensuring that collusive behavior could not possibly be part of an equilibrium. Would anyone be convinced by such an argument? Would we be convinced by a similar argument that fiat money is worthless, and hence refuse to accept it? If so, then games with infinite horizons are out of place. If not, then such models are appropriate. The relevant criterion is not the realism of the model, but its ability to provide insights into the behavior of interest.

The instrumental view complicates game theory. A world of literal descriptions and perfectly rational players is typically more orderly than are approximations of a complicated world filled with people. Consider a version of the prisoners' dilemma, the most studied single game in game theory, formulated as the "push-pull" game by Andreoni and Varian (1999). Alice has an apple, worth 1 to her and worth 3 to Bob. Bob has a banana, worth 1 to him and worth 3 to Alice. Alice and Bob simultaneously decide whether to keep their good (pull, or defect) or give it to the other person (push, or cooperate). It is a strictly dominant strategy for Alice to keep her apple—she is better off doing so no matter what Bob does. The same is true of Bob, of course, and mutual defection is the unique (Nash and indeed strictly dominant strategy) equilibrium in the prisoners' dilemma.

Will people defect in the prisoners' dilemma? In the classical view, yes; this is not only obvious but is a tautology (as Binmore 1994, chap. 3 explains). Under this interpretation, the numbers that appear in the payoff matrix are utilities, derived from a revealed preference analysis of behavior. The fact that larger numbers are attached to defecting than to cooperating indicates the agent in question "derives higher utility from defecting," which under the revealed preference interpretation is synonymous to saying that the agent defects. Asking whether the agent might cooperate is equivalent to asking whether we have gotten the game wrong. If the game is correct, there can be no outcome other than defection.

Things are more complicated under an instrumental view. First, the actions "cooperate" and "defect" are approximations of alternatives that may be much more complex. Cooperation may involve colluding in an oligopoly market or signing a nuclear arms agreement, while defection may involve flooding the market with increased output or installing an antiballistic missile shield. In addition, we typically cannot hope to measure utilities, and the numbers in the cells are instead measures of profits or some other more-readily-measured quantity. Will the players defect? Equivalently, have we chosen well in approximating the interaction as a prisoners' dilemma? This can be a difficult question.

Evolutionary Game Theory

How do we think about equilibrium under the instrumental view? Here, we bring game theory back to long-standing traditions in economic theory. Economic models of individual behavior (the first of the two pillars of methodological individualism) are built around maximization. When presenting the idea of utility maximization or profit maximization in introductory economics classes, one invariably encounters questions as to whether people or firms really maximize, often accompanied by examples of experience with satisficing behavior, cost-plus pricing, or some other behavior that appears to bear no relationship to maximization. A standard response (for example, see Alchian 1950) is that people or firms probably do not literally maximize, but rather they make choices and adjust these choices in light of their experience, sometimes experimenting and sometimes making mistakes, while continually noting which alternatives appear to lead to better outcomes than which others. This gives rise to an adaptive process leading them (at least approximately) to the alternatives that solve their maximization problem. The original economic models of the second pillar, competitive markets, similarly made an appeal to a (sometime implicit) market adjustment process. Walras (1874) not only introduced the notion of competitive equilibrium, but also the *tâtonnement* process that he envisioned leading to such an equilibrium.

Evolutionary game theory applies analogous reasoning to games (for book-length treatments, see Fudenberg and Levine 1998; Samuelson 1997; Sandholm 2010; Vega-Redondo 1996; Weibull 1995; Young 1998). The idea is not that players deduce the equilibrium actions from the structure of the game, or that the equilibrium springs into life upon the appearance of the game. Instead, we think of people as accumulating experience with the game. They choose alternatives, check how well these alternatives work, perhaps experiment with other alternatives, and sometimes make mistakes, all giving rise to a trial-and-error process that (one hopes) tends to push them toward equilibrium. Using phrases reminiscent of other areas in economics, the rational calculations of the classical approach are replaced by the limiting outcomes of an adaptive process. This view brings the methods used in game theory not only closer to that of traditional economics, but also closer to that of the physical sciences. In the latter, it is common that one first specifies a dynamic process, and then views equilibria as the rest points of this dynamic process. Classical game theory is noteworthy in that equilibria come into being divorced from a dynamic process. Evolutionary game theory puts the dynamic process back into the picture. Interestingly, Cournot (1838) motivated his equilibrium for the duopoly illustrated in Figure 1 as the limiting outcome of a best-response-based adjustment process.

Evolutionary game theory was initially surrounded by a great deal of excitement, and like equilibrium refinements, for a while (approximately the 1990s) lay at the center of game theory as well as perhaps economic theory more generally.⁶

⁶For an overview, see the Spring 2002 *Journal of Economic Perspectives* "Symposium on Evolutionary Economics," including Bergstrom (2002), Nelson and Winter (2002), Robson (2002), and Samuelson (2002).

More recently, it has receded into the background. To a large extent, this reflects the success of evolutionary game theory. Evolutionary game theory addressed two basic questions. Can we expect the dynamic processes shaping behavior in games to lead to Nash equilibria? Can we expect them to lead to refinements of Nash equilibrium? We now have a good understanding of these questions.

Broadly speaking, the answer to the first is a statement of the form that “stable outcomes of evolutionary models are Nash equilibria.” More detail would be required to make this summary statement precise, and there are a variety of settings in which it does not hold (see Hofbauer and Sandholm 2011 for a particularly strong example and Vega-Redondo 1996 for an example pertaining to the duopolists of Figure 1). However, precise versions of this result have appeared in a wide variety of models and settings (Samuelson 2002). Consequently, the consensus is that economists working with game theoretic models can devote attention to Nash equilibria confident that, in the appropriate circumstances, there are foundations for this convention.

The answer to the second question is negative—evolutionary models do not consistently lead to any of the standard refinements of Nash equilibrium, much less produce a consensus on what a useful refinement might be. The point of departure for equilibrium refinements is the presumption that players in a game will not select weakly dominated strategies.⁷ For example, in two-player games, refining the Nash equilibrium concept by stipulating that players avoid weakly dominated strategies leads to the concept of a perfect equilibrium, one of the first and most important equilibrium refinements. Elsewhere the relationship between weak dominance and equilibrium refinements is more subtle, but the spirit of weak dominance still permeates the refinements literature. However, evolutionary dynamics do not routinely eliminate weakly dominated strategies (for example, Samuelson 1993).

On the strength of these insights, evolutionary game theory has moved off center stage, while game theory itself remains inextricably woven throughout economics.

Challenges for Game Theory

We discuss here three central challenges facing game theory.

Equilibrium Selection

Games often have multiple equilibria. This can be true of the simplest games, with only two players and only two actions per player. For example, people drive on the right side of the road in some countries and drive on the left in others. The obvious way to model this behavior is as a coordination game. It is natural to think of payoffs that make it a best response for any given player to drive on the

⁷See footnote 3 for a definition of weak dominance.

right as long as others do so, and similarly to drive on the left as long as others do, giving two equilibria (plus a mixed equilibrium in which each driver chooses to drive on the left with probability one-half and to drive on the right with probability one-half, though this mixed equilibrium is presumably not helpful in describing driving behavior).

There is nothing exceptional about this case—many games give rise to multiple equilibria. McLennan (2005) shows that standard normal-form games can have enormous numbers of equilibria, while Ledyard (1986) shows that any undominated behavior can be rationalized as the equilibrium of a coordination game. One need not generalize the Cournot duopoly very far beyond the linear demand and cost functions of Figure 1 to obtain multiple equilibria. Multiple equilibria even more obviously arise in repeated games, such as a repeated version of the Cournot duopoly, where, as noted earlier, a collection of “folk theorems” tell us that if the players in such a game are sufficiently patient, then almost any outcome can be an equilibrium (Fudenberg and Maskin 1986; Mailath and Samuelson 2006).⁸ A common lament is that a theory that predicts anything can happen has no predictive power at all. Continuing in this vein, concern is sometimes expressed that the multiplicity of equilibria renders repeated games useless, if not game theory itself.

Game theory is not alone in giving rise to multiple equilibria. Many other economic models have multiple equilibria, reflected in such notions as liquidity traps or poverty traps, as well as in explanations for the Great Depression as an unfortunate equilibrium in a game with multiple equilibria (Cooper and John 1988). More strikingly, the Debreu–Mantel–Sonnenschein theorem gives us a result analogous to the folk theorem of repeated games: *any* continuous function satisfying linear homogeneity, Walras’ law, and a “boundary condition” (that the quantity demanded of a good explodes as its price goes to zero) is an excess demand function of some economy (Debreu 1974; Mantel 1974; Sonnenschein 1973). Nonetheless, the response has not been a call to abandon the theory of competitive equilibrium, partly because one can find empirical content in general equilibrium models (for example, Brown and Kübler 2008; Brown and Matzkin 1996; Chiappori, Ekeland, Kübler, and Polemarchakis 2004), and partly because the welfare theorems provide useful insights despite the prospect of multiple equilibria.

One possible reaction is that the multiplicity of equilibria is more troubling in the case of game theory, or at least repeated games, than with competitive equilibrium. One often generates multiple outcomes in general equilibrium partly by varying the technology or preferences in the model. A repeated game more readily gives multiple equilibria despite holding the technology and preferences fixed. But this comparison masks other, countervailing differences. Competitive equilibrium models assume that agents perceive themselves to be negligible in the market.

⁸Slightly more precisely, for any payoff profile that is feasible in the stage game and that is individually rational, in the sense that there is no way for either player to guarantee that they receive a higher payoff, if the players are sufficiently patient, then there is an equilibrium of the repeated game giving that payoff.

Repeated games have no counterpart of this convention, and it is no surprise that one can construct more equilibria when given more degrees of freedom.

Multiple equilibria is not the only respect in which game theory has been troubled by an abundance of riches. Game theory seeped slowly into economics after its origins in the work of von Neumann and Morgenstern (1944) and Nash (1950a, b, 1951, 1953), but then the use of game theory in economics exploded in the 1980s. This burst of work began with articles, such as Rubinstein (1982), Kreps and Wilson (1982a, b), and Milgrom and Roberts (1982a, b), that demonstrated the power of (primarily noncooperative) game theory and served as the catalyst for an outpouring of work.

Industrial organization was a natural area of application for this work, and the result was a strategic revolution in industrial organization. A field that had been heavily empirical, seeking relationships between empirical measures of concentration and other structural features of an industry on the one hand and profits or other performance measures on the other, became enthusiastically theoretical. To see this change, one need only compare the Econometric Society World Congress lectures on industrial organization given by Weiss in 1969 (Weiss 1971), Schmalensee in 1980 (Schmalensee 1982), and Roberts in 1985 (Roberts 1987). Strategic models came to be used to explain price discrimination, advertising, entry deterrence, limit pricing, and a host of other phenomena. The difficulty was that an impression soon formed that a sufficiently determined modeler could construct a model explaining any behavior, no matter how counterintuitive. Here was another version of a folk theorem, pertaining not to a specific model such as a repeated game, but to a modeling approach. A common view is that a successful tool must exclude as well as include certain behaviors, and as a result the strategic revolution in industrial organization did not maintain its momentum.

How will we make progress on equilibrium selection in games? One response is to focus on results that depend only on the presumption that *some* equilibrium is chosen, without being specific as to which equilibrium. For example, one reason economists are comfortable with multiple equilibria in a competitive economy is that the first welfare theorem applies to all equilibria. This allows basic results in welfare analysis to be established that do not depend on which of possibly many competitive equilibria might be relevant. At this point, however, game theory has not produced functional equivalents of the welfare theorems.

A second possibility is to let empirical methods point the way to an equilibrium. The emphasis on strategic models in industrial organization has given way to a current emphasis on structural empirical models, as seen in the 2010 Econometric Society World Congress lectures on industrial organization given by Bajari, Hong, and Nekipelov (2013) and Aguirregabiria and Nevo (2013). Papers in this area often consider models that admit multiple equilibria and respond by assuming that the observed behavior reflects *some* equilibrium, and that this same equilibrium is reflected consistently throughout the data. This is typically enough to proceed, with the results providing clues not only about the structure of the game but also about the resulting behavior.

Yet another possibility is to note that in some cases, models with multiple equilibria may provide the best match for the interaction being studied, and we should embrace the multiplicity rather than endeavor to abolish it from our models. In the bank run scene from the 1946 film *It's a Wonderful Life*, George Bailey (played by James Stewart) gives an impassioned speech that a game theorist might reasonably paraphrase as, "There are two equilibria to this game, one in which the bank fails and one in which it survives, and we should endeavor to have the latter." Diamond and Dybvig (1983) capture this intuition in a model that relies crucially on the presence of multiple equilibria. A large subsequent literature has developed this idea.

In many other applications of game theory, however, results hinge on selecting a particular equilibrium for study. Progress in dealing with multiple equilibria will then require taking the instrumental rather than classical view of game theory quite seriously. In the instrumental view, the choice of equilibrium concept, and indeed the choice between multiple equilibria satisfying that concept, is part of the construction of the model, and should be informed by the details of the application one has in mind. If one is modeling an encounter between two agents that have limited experience and knowledge of one another, such as the president of the United States and a Middle Eastern dictator suspected of harboring weapons of mass destruction, a restriction to rationalizable strategies may be too demanding. Instead, one might reasonably question whether the participants are rational, much less whether it is common knowledge that they are rational. On the other hand, we have no difficulty applying Nash equilibrium, and even applying a particular Nash equilibrium, in settings where the participants have enough historical or cultural experience with the game. We take it for granted that people will drive on the left in the United Kingdom and on the right in the United States.⁹ Backward induction is a reasonably reliable expectation when games are played by chess grandmasters, but less so when played by ordinary undergraduates (Palacios-Huerta and Volij 2009).

Suppose the two duopolists of Figure 1 are considering entering a market in which production is costly. Staying out of the market gives either firm a payoff normalized to be zero. The market demand function is such that a firm who is the sole entrant—no matter which one—earns a monopoly profit of one. However if both firms enter, they each lose one. This "entry game" has three Nash equilibria. Two of them are asymmetric but pure equilibria, in which one firm enters and the other firm does not. The third is a symmetric, mixed equilibrium in which each firm enters with probability half. If played by unacquainted strangers with no contextual clues and no asymmetries in the environment or the presentation of the game, perhaps in a laboratory, we would have no reason to expect one of the pure equilibria to appear. If play is to resemble an equilibrium, it will have to be the mixed equilibrium. However, when studying entry decisions on the part of two

⁹This observation may seem trivial, but it does not arise out of Harsanyi and Selten's (1988) analysis of equilibrium selection under the classical view of game theory, which denies itself any help that might come from the context in which the game is played.

firms, we might much more reasonably expect a pure equilibrium. In the latter case, the actual entry process is complicated and dynamic, and some institutional details—one firm got there first, or has cost synergies arising out of a better developed distribution network, or something else—will allow the firms to coordinate on what looks like a pure equilibrium in the static model. Which pure equilibrium? Like the distinction between the mixed and pure equilibria, this depends on details that have been left out of the game, but that should be incorporated in the model by choosing an equilibrium in light of the modeler's study of the institutional details.

These institutional details will typically involve elements of history that have been omitted from the description of the game. The model of the game makes no mention of whether the players are encountering the game for the first time, or have a personal or cultural history with the game. The strategies in the mathematical representation of the game are typically given neutral labels. In practice these strategies correspond to actions that are interpreted in light of the personal and cultural context of the game. As Schelling (1960) vividly illustrates, this context can have an almost magical effect in distinguishing between the various equilibria of the game.¹⁰ Classical game theory views the specification of the model as straightforward, and focuses on what to do once one has the model. Under instrumental game theory, the specification of the model takes center stage. This specification requires an understanding of the application, setting, and history of the game, all of which should inform the specification not only of the game but also of the choice of equilibrium concept and equilibrium.

Considerable work remains to be done on identifying both which equilibrium concept we should be using and which of the potentially many equilibria consistent with that concept should command our attention. John Maynard Keynes remarked (in a letter to Roy Harrod written in 1938, reproduced as letter 787 in Besomi 2003) that, "Economics is the science of thinking in terms of models joined to the art of choosing models which are relevant to the contemporary world." Graduate courses in economics tend to focus on the science of working with models. Progress on equilibrium selection will come from careful work on the art of choosing models. This is a joint choice involving both the game and the relevant equilibrium, and will typically depend on the setting to which the analysis is to be applied.

¹⁰Schelling (1960) introduces the concept of a focal point, to capture the idea that the context in which a game is played often affects expectations and behavior in the game. For example, Schelling considers the case of two people who have fixed a day to meet in New York City, but have said nothing about either the time or location. The abstract representation of the game provides no way to distinguish different times and locations, and hence no hope that the two will actually meet. Schelling reports that in informal surveys, nearly every respondent indicated that they would attempt to meet at noon, and a majority chose Grand Central Station (perhaps reflecting the time in which the book was written) as a location. A large literature has subsequently grown around the idea of focal points (for example, Binmore and Samuelson 2006), but there is still much work to be done, as is reflected in Schelling's remark (at the Arne Ryde Conference held in his honor in Lund, Sweden on August 23, 1997) that the theory of focal points has done more for game theory than game theory has done for the theory of focal points.

Applications

Game theory is much more successful in some applications than others. Two of the obvious successes of game theory are auctions (for example, Klemperer 2004; Krishna 2002; Milgrom 2004) and matching (for example, Roth 2008a, b, 2015). Research in these areas has produced not only a rich body of new theoretical results, but has also transformed the way that resources are allocated in a wide variety of markets. Resources that governments used to give away are now routinely auctioned, with significant implications for government revenue and, perhaps more importantly, for the efficient allocation of resources. Auctions have become a common mechanism for firms to price their products, including relatively new products such as online advertising opportunities, but also quite familiar products such as electricity. New entrants in a variety of professional fields are now allocated to employers via matching algorithms, as are students allocated to schools. As applied to the “market” for matching suitable kidney donors to recipients, one could argue that matching theory has saved thousands of lives.

The result has been a flourishing new field of market design, which might be described as the application of game theoretic models, insights, and intuition to the solution of practical resource allocation problems. Economists are fond of market tests, and in the form of market design, game theory has clearly passed a market test of its usefulness. Designing markets and advising the participants in these markets now presents brisk employment opportunities for game theorists.

Other applications of game theory have been less successful in such market tests, with perhaps the leading example being the theory of bargaining. Edgeworth (1881, pp. 20–30) identified the bargaining problem as the basic point of departure for studying economics. Nonetheless, for decades afterwards, it was common to say that we might expect bargaining outcomes to be efficient, but that economic reasoning was not helpful in identifying which of the typically many efficient outcomes might appear. Game theory staked its claim to bargaining early. Two of Nash’s quartet of early papers (Nash 1950b, 1953) addressed the bargaining problem. Rubinstein’s (1982) analysis of bargaining played a role in spurring the use of game theory in economics. A new literature on bargaining followed (for a summary, see Muthoo 1999). However, game theory has not had the same success in bargaining that it has had with auctions or matching. One cannot easily point to examples where bargaining methods have been overhauled in response to game theoretic insights. Game theorists routinely bring game theoretic models to bear when called upon for advice concerning auctions and matching processes, but are less likely to appeal to game theoretic models when consulted on bargaining problems.

A common concern about game-theoretic models of bargaining is that the outcomes are too sensitive to fine details of the model. The timing of offers and counteroffers, the specification of the information structure, the length of the horizon, the length of a time period, and other details can all have an important effect on the outcome. For a striking example, Shaked (1994) shows that it can make an important difference whether one party to the bargaining process can quit the game *before* or *after* hearing a counteroffer from the other side. It is seldom clear

when examining an interaction between a union and a firm or between political groups in the Middle East whether we should model this as a discrete or continuous time game, a rigidly alternating offers game or a game in which either side can make an offer at any time, and so on. Given a choice from a collection of models that give sharply different results, with little guidance as to which is appropriate, it is not surprising that one might avoid using such models.

This explanation for the limited role of game theoretic analysis in bargaining does not tell the whole story. Modeling an auction also gives rise to a seemingly endless series of choices—are values common or idiosyncratic, are the bidders risk neutral or risk averse, is there a resale market, will the bidders collude, are the bidders symmetric, and so on—again without definitive indications as to which is the obvious modeling choice. The difference appears to be a belief that auction models have come sufficiently close to isolating the essential features of an auction, and auction theorists have sufficiently honed their intuition in the course of working with such models, that the models are useful tools in designing, running, and participating in auctions. It is less clear that bargaining models have isolated the essential features and allowed us to sufficiently hone our intuition. The basic tradeoff in a model of a first-price auction is clear—shading one's bid below one's valuation makes the outcome more lucrative if one wins, but makes one less likely to win. This appears to be a first-order effect in many auctions. The basic feature highlighted in most bargaining models is patience, with less-patient people being in a weaker bargaining position. It is less obvious that patience is typically a first-order effect in bargaining.

As is the case with economic theory, much of the progress in game theory comes not from the science of applying models, but from the art of formulating them. Game theory has hit upon extraordinarily useful models in some areas, but has been less successful in others. One hopes this means that there are still important discoveries ahead for game theory.

Cooperative Game Theory

Noncooperative game theory assumes that players act independently, with the central question being whether a player can gain from a unilateral deviation. Cooperative game theory assumes that players can form coalitions, with the central question being whether a collection of players can find a (binding) allocation of the payoffs available to the coalition that would allow them all to gain from forming the coalition. Figure 1 is an example of the noncooperative approach to a Cournot duopoly, which assumes the two firms choose their outputs simultaneously and independently. A cooperative approach would recognize that both firms could do better by forming a coalition and splitting the resulting monopoly profits.

The early work in game theory was dominated by cooperative game theory: for example, von Neumann and Morgenstern's (1944) *Theory of Games and Economic Behavior* devoted much of its attention to cooperative games. One of the fundamental results in general equilibrium theory, the Debreu–Scarf theorem (Debreu and Scarf 1963), offering a precise version of the intuition that a large economy should be essentially a price-taking economy, is based on the (cooperative) notion

of the core.¹¹ The Shapley value (Shapley 1953) has proven to be useful in studying the power of political coalitions (as in Winter 2002) and more practically in solving cost-allocation problems (Young 1994).¹² The “nucleolus” (Schmeidler 1969) has attracted attention for (among other things) unexpectedly turning out to explain a bankruptcy prescription from the Talmud (Aumann and Maschler 1985), solving a problem that had remained the subject of discussion and contention for millennia.¹³ There are many other examples.

More recently, cooperative game theory appears to have disappeared from economics. First-year graduate theory courses routinely cover the basics of noncooperative game theory, but may not even mention the core. The classic texts that shepherded game theory into widespread use in economics, Fudenberg and Tirole (1991) and Myerson (1991), are weighted toward the discussion of noncooperative game theory. How can game theory avoid losing the tools and insights of cooperative games?

We can go back to the beginnings for the answer. Nash (1953) introduced what has since come to be called the “Nash program.” In this paper, Nash presented a noncooperative bargaining game whose outcomes coincided with the Nash bargaining solution (an axiomatically motivated rule for sharing a surplus) that he had introduced earlier (Nash 1950b). Work in a similar vein has subsequently provided noncooperative foundations for the core (Perry and Reny 1994), the Shapley value (Gul 1989), and the nucleolus (Shubik and Young 1978). The idea of the Nash program is that one should combine the cooperative and noncooperative approaches. Our understanding of a cooperative solution concept is bolstered by examining the noncooperative games that lead to such a concept. Our understanding of noncooperative games is bolstered by a cooperative characterization of their outcomes. Both directions are important. We understand best and can most usefully apply concepts that can be given both cooperative and noncooperative foundations.

Recent work in matching has brought the Nash program back into the mainstream (for an introduction, see Roth and Sotomayor 1990). The basic equilibrium concept in this literature is that of a stable match. When matching students to schools, for example, stability requires that there should be no student and school who are currently unmatched and who have the property that the student prefers the school to her current match and the school would rather have the student than one of their existing students (or an empty seat). This is a cooperative equilibrium concept, closely related to the idea of the core, as it revolves around the requirement that there be no two-person blocking coalitions (with larger coalitions coming into play in more complicated matching environments). However, the standard

¹¹ A specification of payoffs for the players in a game is in the core of that game if there is no “blocking” coalition that can form and distribute its payoffs in such a way as to make all of its members better off.

¹² Intuitively, the Shapley value is a specification of payoffs that allocates to each player in a game the average of the marginal contributions that the player makes to the various coalitions that might form.

¹³ Intuitively, the nucleolus is the allocation in which the coalition that is least happy about its share of surplus is at least as happy as possible.

route to proving the existence of a stable allocation is to construct a noncooperative process, such as the deferred acceptance algorithm, that leads to stable allocations (Gale and Shapley 1962).¹⁴ Indeed, the deferred acceptance algorithm lies at the heart of the processes used to implement outcomes in the many matching markets that have recently been transformed by applications of game theory. We are left doubly confident of the procedure, with the cooperative lens confirming that it leads to outcomes with attractive properties and the noncooperative lens ensuring that one gets there via an intuitive procedure.

The Nash program thus holds out the promise of combining the best of cooperative and noncooperative game theory. However, considerable work remains if we are to realize the potential of this approach. In some cases, it will require new work in cooperative game theory. Noncooperative game theory has been especially fruitful in examining problems of incomplete information, an area in which cooperative game theory has not been particularly active (for a recent step in this direction, see Liu, Mailath, Postlewaite, and Samuelson 2014). In other cases, as Binmore, Rubinstein, and Wolinsky (1986) point out, implementing the noncooperative component of the Nash program requires considerable care. In yet other cases, the approach has not produced immediate gains. For example, one can view the research on “renegotiation proofness” as an attempt to use cooperative ideas, namely that players should not be content with a continuation equilibrium whose payoffs are Pareto dominated by an alternative continuation equilibrium (and hence is blocked by a coalition of the whole), to select equilibria in a repeated game. Applications of renegotiation proofness have been hindered by the specter of multiple notions of renegotiation proofness. For an introduction to this issue, see Mailath and Samuelson (2006, Chapter 4.6); for a recent alternative perspective, see Miller and Watson (2013).

Prospects and Directions

Game Theory beyond Economics

We live in what might be called the imperial age of game theory, in which game theory has become influential in an ever-growing variety of other disciplines. Game theory is now a standard tool in political science. For example, McCarty and Meirowitz (2007) provide a book-length overview of how game theory can be used to examine the relations between countries, the behavior of political parties, electoral behavior, the workings of legislatures, lobbying, and so on. For an earlier

¹⁴ For example, consider a male–female marriage market and the “men-proposing” version of the deferred acceptance algorithm. Each unmarried man proposes to the woman who is his first choice. Each woman who receives at least one proposal holds her most-preferred proposal and rejects the rest. In the next round, each man who has been rejected then proposes to his top preference among the women to whom he has not yet proposed. A woman who receives proposals in this round holds her most-preferred proposal, whether it is a new one or one held from the previous round, and rejects the rest. Successive rounds follow. In a finite number of rounds, this process reaches a stable allocation, at which point no more offers are made, and the current portfolio of held proposals is accepted.

application of game theory insights to political economy issues, economists will of course think of the work of Elinor Ostrom (for example, 1990) on governing a commons. Ostrom's graduate training was in political science, although she shared the Nobel Prize in economics in 2009. Game theoretic analysis is now common in law: for book-length overviews, see Baird, Gertner, and Picker (1994) and Zaluski (2015). Game theory appears in the study of philosophy, especially ethics, with Binmore (1994, 1998) on *Game Theory and the Social Contract*, Gauthier (1986) on the foundation of moral norms, and Skyrms (2004) on the evolution of social structure offering good examples. Psychologists, especially experimental psychologists, have turned to game theory, with Colman (1999) presenting an overview of *Game Theory and Its Applications in the Social and Biological Sciences*, as have neuroscientists (for example, Glimcher, Camerer, Fehr, and Poldrack 2009). Perhaps the greatest empirical success of game theory is in biology, with Maynard Smith and Price (1973) and Maynard Smith (1982) being the obvious point of departure, while Broom and Rychtář (2013) provide a more recent book-length overview of *Game-Theoretical Models in Biology*. Hammerstein and Riechert (1988) is a particularly striking application of a game-theoretic analysis of two populations of desert spider. Under the guise of algorithmic game theory, game theory has spread throughout computer science (for example, Nisan, Roughgarden, Tardos, and Vazirani 2007). Game theory has become a standard tool in electrical engineering, as seen in Lasaulce and Tembine's (2011) *Game Theory and Learning for Wireless Networks*. Game theory has been influential in operations research, as seen in the prevalence of game theory papers in *Operations Research* and *Mathematics of Operations Research*, and has moved into other areas of business schools, such as accounting and marketing.

As remarkable as the breadth of the disciplines that have felt the reach of game theory is the breadth of agents who have appeared in game theoretic models. Behind the rather bland moniker of "players" one can obviously find people, but can also find firms, unions, political parties, and countries. One can find parts of people, in the form of cells and neurons. One can find plants and animals stretching from the most intelligent to the lowliest of microorganisms. One can find mechanical devices, such as switches and routers in distributed information-processing systems. Notice that many of these interpretations of a player are clearly incompatible with a classical conception of a player as a rational actor, or with the view of a player being able to deduce the equilibrium implications of a game. As game theory has spread beyond economics, it has necessarily moved every more firmly into the instrumental camp. Game theory appears to be on its way to becoming not just the language of economics, but the language of the social sciences, and perhaps beyond.

A New Home for Game Theory?

The first hints of game theory appeared in mathematics, including Cournot's (1938) analysis of duopoly, Zermelo's (1913) examination of finite games of perfect information, and Borel's formulation of the notion of a strategy and of zero sum

games (Dimand and Dimand 1992, pp. 18–23).¹⁵ When game theory was established as a field of study in the 1940s and early 1950s, it was pursued primarily within mathematics (for an accessible history of game theory, see Leonard 2010).

The situation is now reversed. It is a rare department of mathematics that offers a course in game theory. In contrast, every first-year graduate sequence in economic theory includes a substantial amount of game theory, and undergraduate and graduate game theory courses are common in economics departments. This shift from associating game theory with mathematics to economics is reflected in the journals. Following the appearance of the *International Journal of Game Theory* in 1971, the next journal devoted to game theory was *Games and Economic Behavior*, first appearing in 1989 and building the link to economics into its title.

Should game theory be housed in economics? The answer is not obvious. On the one hand, the interaction between economic applications and basic results in game theory has been particularly fruitful. Many of the advances in game theory have been motivated by particular economic applications. The study of duopoly in Cournot (1838) gave rise to a precursor of the idea of Nash equilibrium. Later, von Stackelberg's (1934) examination of duopoly gave rise to a precursor of backward induction. Friedman's (1971) study of collusion gave rise to a precursor of the folk theorem. Edgeworth's (1881, pp. 34–56) consideration of competitive markets gave rise to an early counterpart of the core. Such interactions may be sufficiently fruitful that game theory is most efficiently studied within economics.

On the other hand, as we have noted, game theory is becoming increasingly influential in a variety of disciplines beyond economics. It is presumably inefficient to have a community of people within each discipline working independently on related problems in game theoretic techniques. There may be economies of scale to be gained from establishing departments of game theory. If one is convinced of the virtues of interdisciplinary work and inclined to create new academic structures to foster such work, then game theory may be a prime target.

The Future

What does the future hold for game theory in economics? It seems a safe bet that game theory will continue to be the language of economics. It appears to be an equally safe bet that new areas of inquiry in economics will both make use of game theory and leave their mark on game theory. For example, one of the most striking recent developments in economics has been the advent of behavioral economics (for example, Cartwright 2014). From its beginnings, behavioral economics has provided new ground for the application of game theory. Strotz (1955–56) noted that dynamic utility maximization problems could give rise to inconsistencies, with a single person at different times effectively being different selves with diverging interests. Game theory provides the tools to examine the interaction between these

¹⁵Borel wrote a series of papers on game theory between 1921 and 1927. The most accessible point of entry for these papers is *Econometrica's* publication of three of them, translated by Leonard Savage and with a commentary by John Von Neumann (Borel 1953a, b, c).

multiple selves. However, behavioral economics has much more to offer beyond multiple selves, and the new behavioral insights are finding their way into “behavioral” game theory (Camerer, Loewenstein, and Rabin 2004; Camerer 2003). Once again, we see a fruitful interaction between economics and game theory, with each leaving its mark on the other.

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