Examen Teoría de Juegos. Febrero 2016. Solución

1. No hay equilibrios en estrategias puras. El único equilibrio es en el que los dos jugadores randomizan con probabilidades $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$.

   To show that this NE is unique, all we need to do then is to show that there cannot be a NE in which one of the players play a strategy where the support consists of exactly two actions. Suppose this is the case; in particular, suppose we have a NE in which player 2 randomizes over S and P. Then, player 1 cannot be playing P with positive probability. Hence, player 1 must be randomizing over R and S (since we have already argued that there is no NE in which any player plays a pure strategy). But if player 1 is playing both R and S with positive probability, player 2 cannot be playing P with positive probability - i.e. we have a contradiction.

2. *a*) For every $e \in [0, 1]$, $(e, \ldots, e)$ is a Nash equilibrium.

   b) Any symmetric equilibrium with $e < 1$ is Pareto inefficient, because all the players would be better off if they simultaneously switched to $(1, 1, \ldots, 1)$. On the other hand, the symmetric equilibrium $(1, 1, \ldots, 1)$ is Pareto efficient.

   c) El equilibrio simétrico $(\frac{1}{2}, \frac{1}{2}, \ldots, \frac{1}{2})$.

3. The following Bayesian game models the situation.

   Players: The two people.

   States: The set of states is {strong, weak}.

   Actions: The set of actions of each player is {fight, yield}.

   Signals: Player 1 receives the same signal in each state, whereas player 2 receives different signals in the two states.

   Beliefs: The single type of player 1 assigns α to the state strong and probability $1-\alpha$ to the state weak. Each type of player 2 assigns probability 1 to the single state consistent with her signal.

   Payoffs: The players' Bernoulli payoffs are as follows:

'strong'	fight	yield
fight	-1, **1***	1, 0
yield	0, **1***	0, 0

'weak'	claim	not
claim	1, -1	1, **0***
not	0, **1***	0, 0

The best responses of each type of player 2 are indicated by asterisks. Thus if $\alpha < \frac{1}{2}$; then player 1's best action is fight, whereas if $\alpha > \frac{1}{2}$ her best action is yield. Thus for $\alpha < \frac{1}{2}$ the game has a unique Nash equilibrium, in which player 1 chooses fight and player 2 chooses fight if she is strong and yield if she is weak. When $\alpha > \frac{1}{2}$ the game has a unique Nash equilibrium, in which player 1 chooses yield and player 2 chooses fight regardless of whether she is strong or weak.

4. *a*) Consider the following strategy profile $(s_1; s_2)$:

$$s_1\left(h^t\right) = \begin{cases} B & \text{if } h^t = h^0 \\ T & \text{if } h_1 = (B, R) \\ M & \text{otherwise} \end{cases}$$

$$s_2\left(h^t\right) = \begin{cases} R & \text{if } h^t = h^0 \\ L & \text{if } h_1 = (B, R) \\ C & \text{otherwise} \end{cases}$$

It is easy to check that $(s_1; s_2)$ is a SPE of the repeated game. In any subgame of the second period the players play a Nash equilibrium of the stage game. Player 2 does not have any incentive to deviate in the first period. For player 1 is concerned, if she follows the equilibrium strategy her payoff is 7. By deviating she can get at most a payoff of 6.

b) Repitiendo el razonamiento anterior se tiene que el jugador 1 no se va a desviar sii $4 + 3\delta \geq 5 + \delta$, lo que es equivalente a $\delta \geq \frac{1}{2}$.